## 2.3 A brief summary of Stern-Gerlach experiments

- The physical picture of the experiment gets translated mathematically to the set of vectors: $\mathbf{S}_{\mathbf{z}}^{+}$, $\mathbf{S}_{\mathbf{z}}^{-}$, $\mathbf{S}_{\mathbf{x}}^{+}$, $\mathbf{S}_{\mathbf{x}}^{-}$, etc.
- The space represented by these vectors is *an abstract two-dimensional space*. Note: the axes in this abstract space: $\mathbf{S}_{\mathbf{z}}^{+}$, $\mathbf{S}_{\mathbf{z}}^{-}$, for example, are not the same axes where the magnetic fields are aligned.
- This abstract space is a portion of what we will later begin to describe as the Hilbert space.
- It appears that the "projection" process we discussed in this space completely describes what happens in the measurements involved during Stern-Gerlach experiments.
- But, this space is complex. Hence, it is difficult to properly draw it on a board (because it is two-dimensional *and* complex). However, we may be able to visualize it in our minds.
- Certain words are now limiting because of the presence of complex numbers. The words, angle, or dot product were originally developed for real-space vectors and we will need to generalize these definitions.

 

- We find that the following represents a mathematically consistent description of what is happening in these experiments:

  Stern-Gerlach spin states $\rightarrow$ vectors (in a complex space).
  Measurement $\rightarrow$ projection
  *Note also that the projection appears to occur onto special directions!!*
  Projected components $\rightarrow$ complex numbers in general
  Measured quantities $\rightarrow$ absolute values of projected components.

  Hence, although the projected components may be complex, the measured values are generally real.

- While we have based our study thus far on the Stern Gerlach experiments, which deal with spin, we will find later that the above description is appropriate for *all kinds of measurable quantities*.
  - For example, we will late introduce an arbitrary vector that describes the electronic properties of a system and even that vector resides in the complex Hilbert space we described above.
  - An experiment conducted on that vector is a projection similar to what we have introduced above.
- The above ideas will, in a few classes, translate to "the postulates of quantum mechanics".