18.4 When would you need Angular momentum addition and Clebsch Gordon coefficients?

- We have seen how one can transform eigenstates of the operators $\left\{\mathbf{J}_1{}^2, \mathbf{J}_{1z}, \mathbf{J}_2{}^2, \mathbf{J}_{2z}\right\}$ to eigenstates of the operators $\left\{\mathbf{J}^2, \mathbf{J}_z, \mathbf{J}_1{}^2, \mathbf{J}_2{}^2\right\}$.
- That is, we have concerned ourselves with how one might transform the $|j_1, m_{j_1}, j_2, m_{j_2}\rangle$ states to $|J, M, j_1, j_2\rangle$.
- As noted in the section dealing with angular momentum theory, the operators $\mathbf{J}_i$ above are generic angular momentum operators and may represent spin-, orbital- or any combination including L-S coupling.
- The reasons behind why we need to worry about angular momentum addition have been discussed in detail earlier in the notes, but we will restate it here for easier reference.
- The total Hamiltonian of a system does depend on the angular momentum. We will see this is some detail when we discuss hydrogen atom. Essentially, the angular momentum adds a term to the total Hamiltonian that reflects angular kinetic energy.
- If the physical systems that are depicted by $\mathbf{J}_1$ and $\mathbf{J}_2$ *do not* interact with each other then the system Hamiltonian contains angular momentum as a sum of the individual parts: $\mathbf{H} = \mathbf{A} + \mathbf{J}_1{}^2 + \mathbf{J}_2{}^2$, where A is an operator that does not depend on angular motion.
  - In this case, $\left\{\mathbf{H}, \mathbf{J}_1{}^2, \mathbf{J}_{1z}, \mathbf{J}_2{}^2, \mathbf{J}_{2z}\right\}$ commute with each other and the states $|j_1, m_{j_1}, j_2, m_{j_2}\rangle$ may also be considered eigenstates of the system Hamiltonian.

- If the physical systems that are depicted by $\mathbf{J}_1$ and $\mathbf{J}_2$ *do* interact with each other then the system Hamiltonian contains the *total angular momentum*, $\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2$. That is $\mathbf{H} = \mathbf{A} + \mathbf{J}^2$, which will lead to a coupling term $\mathbf{J}_1 \cdot \mathbf{J}_2$.
  - In this case, $\left\{\mathbf{H}, \mathbf{J}^2, \mathbf{J}_z, {\mathbf{J}_1}^2, {\mathbf{J}_2}^2\right\}$ commute with each other and the states $|J, M, j_1, j_2\rangle$ may also be considered eigenstates of the system Hamiltonian.
  - Here the Clebsch-Gordon coefficients provide us an approach to transform from the *convenient* $|j_1, m_{j_1}, j_2, m_{j_2}\rangle$ basis to the $|J, M, j_1, j_2\rangle$ basis.

