Chapter 6

6.2 The standard normal distribution is associated with a normal random variable that has a zero mean and standard deviation of one. But normal random variables can have any mean or positive valued standard deviation. Any normal random variable can be mapped into a standard normal using a transformation that does not alter the probabilities because both a standard normal random variable and any other normal random variable have bell-shaped distributions for which approximately 68 percent in within one standard deviation of the mean.

6.4 a. $P(z>b)=0.9946$; $1-P(z<b)=0.9946$; $P(z<b)=0.0054$; $b = -2.55$

b. $P(z<b) = 0.8186$; $b = 0.91$

c. $P(z>b)=0.0003$; $1-P(z<b)=0.0003$; $P(z<b)=0.9997$; $b = 3.43$

d. $P(-b<z<b) = 0.673$; $P(0<z<b) = 0.3365$;
$P(z<b) = 0.8365$; $b = 0.98$

e. $P(z<b) = 0.0228$; $b = -2.00$

f. $P(0<z<b) = 0.1664$; $P(z<b) = 0.6664$; $b = 0.43$

Using EXCEL, the above z values where obtained as following:

	A	B	
10	0.0054	-2.54909537	=NORMSINV(A10)
11	0.8186	0.910042672	=NORMSINV(A11)
12	0.9997	3.431923687	=NORMSINV(A12)
13	0.1635	-0.980173809	=NORMSINV(A13)
14	0.0228	-1.999078449	=NORMSINV(A14)
15	0.3336	-0.429994316	=NORMSINV(A15)

6.6 *a*. $P(y>b) = 0.0007$; $P(z>\frac{b-25}{9.0}) = 0.0007$;

$$P(z<\frac{b-25}{9.0}) = 0.9993;\ \frac{b-25}{9.0} = 3.20;\ b = 53.8$$

b. $P(y>b) = 0.0550$; $P(z>\frac{b-25}{9.0}) = 0.0550$;

$$P(z<\frac{b-25}{9.0}) = 0.9450;\ \frac{b-25}{9.0} = 1.60;\ b = 39.4$$

c. $P(y<b) = 0.9948$; $P(z<\frac{b-25}{9.0}) = 0.9948$;

$\frac{b-25}{9.0} = 2.56$; $b = 48.04$

d. $P(b>y>25) = 0.032$; $P(\frac{b-25}{9.0}>z>\frac{25-25}{9.0}) = 0.032$;

$P(\frac{b-25}{9.0}>z>0) = 0.032$; $\frac{b-25}{9.0} = 0.08$; $b = 25.72$

e. P(b<y<25) = 0.5429; there is no such value of b because the probability of y less than its mean cannot exceed 0.5.

f. $P(y<b) = 0.2123$; $P(z<\frac{b-25}{9.0}) = 0.2123$;

$P(\frac{b-25}{9.0}<z<0) = 0.2877$; $\frac{b-25}{9.0} = -0.8$; $b = 17.8$

Using EXCEL, the above values can be determined as follows:

	A	B	
24	0.9993	53.75254722	=NORMINV(A24,25,9)
25	0.945	39.38372237	=NORMINV(A25,25,9)
26	0.9948	48.06005626	=NORMINV(A26,25,9)
27	0.532	25.72268335	=NORMINV(A27,25,9)
28	0.5429	No way	
29	0.2123	17.81380724	=NORMINV(A29,25,9)

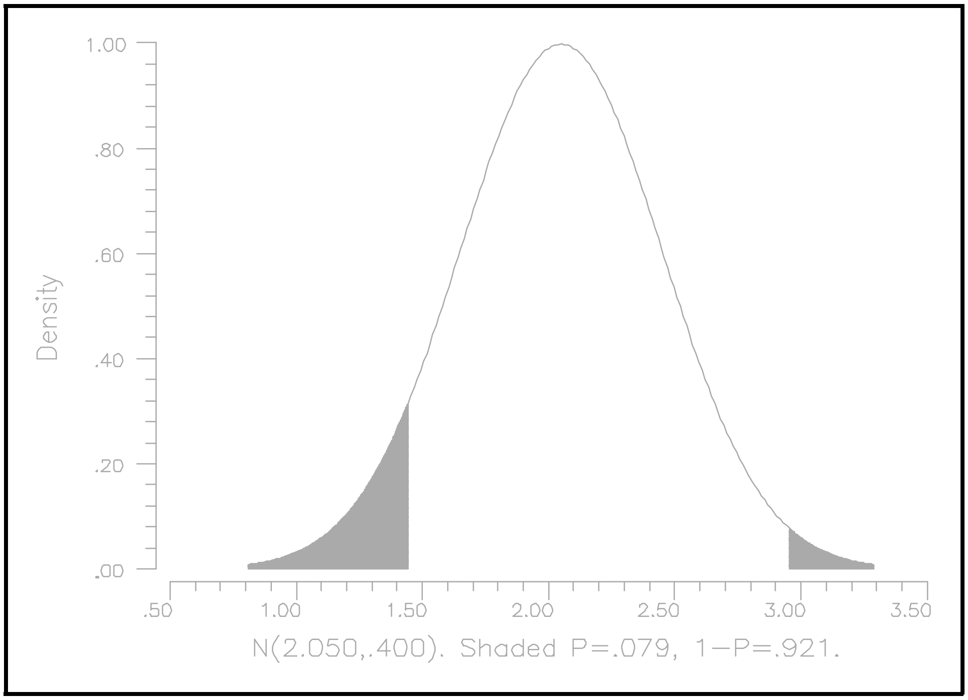
6.8 P(1.45<x<2.95) is required to calculate where X, denoting the completion time, is a normal random variable with a mean of 2.05 and a standard deviation of 0.4.

$$P(1.45<x<2.95) = P(\frac{1.45-2.05}{0.4}<z<\frac{2.95-2.05}{0.4})$$

$$= P(-1.5<z<2.25) = 0.9210$$

A randomly selected employee will take between 1.45 minutes and 2.95 minutes with a probability of 0.921. In the following diagram, the unshaded area identifies the probability of 0.921. Or by using EXCEL we have:

	A	B			
33	1.45	0.066807	=NORMDIST(A33,2.05,0.4,1)		
34	2.95	0.987776	=NORMDIST(A34,2.05,0.4,1)	0.920968	=B34-B33



6.10 It does not seem to be reasonable to assume that the distribution is normal because the distribution is highly right skewed with a very large value pulling up the mean. To be a normal distribution, the mean should be the same as the median which is approximately $10. With the mean of $10, $10 is the distance between the mean and the absolute end point of a zero. However, the distance between the mean and the largest value of $25,000 is $24,990. The distribution cannot be symmetric.

6.12 The probability of an overcharge is 0.90, P(x>0) = 0.9, and the mean overcharge is $223. The probability of not being overcharged is 0.1, P(x≤0) = 0.1, and if the mean undercharge is $25, then the mean of the over, under or correct charge distribution, X, is $198.20(= (0.9)(223) + (0.1)(-25)). If X is normally distributed its standard deviation is then

$154.60, because P(z<-1.282) = 0.1, and $-1.282 = \frac{0-198.2}{F}$

6.14 If X ~ N(198.2, 154.6), and given that an agent is overcharged, the probability that the agent is overcharged by more than $585 is less than 0.01, which is extremely unlikely. We typically do not observe unlikely events, so given that the agent is truthful, either the assumption of normality or the assumed $25 undercharge mean must be called into question. Furthermore, as those who know calculus and computer programs like MATHEMATICA and MAPLE V can attest, if X ~ N(198.2, 154.6), then E(X|X<0) = -73, and the E(X|X>0) = 228, as seen with the following MAPLE V program:

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> h:=1/sqrt(2*Pi)*exp(-½*z^2);
> H:=int(h,z=-infinity..-1.282);
> 198.2+154.6*int(z*h,z=-infinity..-1.282)/H;
> F:=int(h,z=-1.282..infinity);
> 198.2+154.6*int(z*h,z=-1.282..infinity)/F;
```

Thus, there is an inconsistency in assuming the mean of undercharge is $25 and the undercharge mean of $73 implied by X ~ N(198.2, 154.6).

6.16 P(70<x<90) is required to calculate where X, denoting the score, is a normal random variable with a mean of 75 and a standard deviation of 10.

$$P(70<x<90) = P(\frac{70-75}{10}<z<\frac{90-75}{10}) = P(-0.5<z<1.5) = 0.6247$$

That is, 62.47 percent of exam scores are between 70 and 90.

6.18 6 is 3.75 standard deviations below the mean of 36(that is, 6 = 36-3.75×8). Using the rules of normality, we know that almost no values are below μ-3F. -3.75 is smaller than -3. Thus, almost no values are below 6.
Or we can formally calculate P(x<6) where X, denoting length of the time until service is needed, is a normal random variable with a mean of 36 and a standard deviation of 8.

$$P(x<6) = P(z<\frac{6-36}{8}) = P(z<-3.75) = 0.0001$$

6.20 It is given to us that 0.35 ≤ P(6<x<10) ≤ 0.40, where X, denoting women's clothing size, is a normal random variable with 8 as its mean. Normality of X suggests that the following should be satisfied:

$$P(6<x<10) = P(\frac{6-8}{F}<z<\frac{10-8}{F}) = P(-\frac{2}{F}<z<\frac{2}{F})$$ is between 0.35

and 0.40 where F is the standard deviation of the distribution of the clothing size.

That implies $0.455 < \frac{2}{F} < 0.525$ if we find corresponding

z-values in the standard normal distribution table. Thus, the implied value of F should lie between 3.8 and 4.4.

6.22 It is required to calculate P(x>100) where X, denoting days worked to pay taxes, is a normal random variable with a mean of 81 and a standard deviation of 10.

$$P(x>100) = P(z>\frac{100-81}{10}) = P(z>1.9) = 0.0287$$

i.e., the probability of selecting a taxpayer who worked more than 100 days to pay taxes is 2.87 percent.
1 -NORMDIST(100, 81, 10, TRUE) = 1 - 0.047128 = 0.02872

6.24 $P(x>10) = P(z>\frac{10-7}{5.56}) = P(z>0.54) = 0.2946.$

A randomly selected university president will stay in office more than ten years with a probability of 0.2946.

6.26 Under the assumption of normality of the distribution of the average line speed(X), the following probability calculation can be done:

$$P(11250<x<11750) = P(\frac{11250-11500}{142}<z<\frac{11750-11500}{142})$$

$$= P(-1.76<z<1.76) = 2(0.4608) = 0.9216$$

Because the line speed is a continuous random variable that can be expected to vary symmetrically around its mean, with values closer to the mean occurring most often, it seems reasonable to assume that the average line speed(X) is normally distributed.

6.28 Using EXCEL, 1.880789569 =-NORMSINV(0.03); thus, P(z>1.881)=0.03 and the standard deviation of snowfall is 0.53 inches [i.e., 0.531691592 =(3-2)/1.88078957].

6.30 a. IQ is distributed as a normal random variable.
b. The expected value of IQ is 100.
c. The standard deviation of IQ is 15.
d. About 16 percent, or using EXCEL
0.15865526 =1-NORMDIST(115,100,15,1)