Chapter 5

5.2 $P(X>10|n=12,\pi=0.5) = 0.019287$ = 1-BINOMDIST(9,12,0.5,1)

5.4 From the given information, $\pi=0.1$ and $n=10$ where π is the probability of a Cadillac having a defective paint job. Thus, letting X be the number of Cadillacs having defective paint jobs, $P(X<4|n=10,\pi=0.1) = P(X=0) + P(X=1) + P(X=2) + P(X=3) = 0.987204802$ =BINOMDIST(3,10,0.1,1)

5.6 If $\pi = 0.8$ and $n=5$, where π is the probability of a man using a hair dryer, then $P(X=4|n=5,\pi=0.8) = 0.4096$ =BINOMDIST(4,5,0.8,0). We have to assume that five male friends should be a random sample from the large population of American men who use a hair dryer.

5.8 $\pi = 0.95$ and $n=9$ are given where π is the probability that a jurist will uphold a guilty verdict. Thus, letting X be the number of jurists who will do so, $P(X\geq5|n=9,\pi=0.95) = P(X=5) + P(X=6) + P(X=7) + P(X=8) + P(X=9) \approx 1.00$.
We have to assume that each member independently decides to uphold a guilty verdict, which does not seem to be a reasonable assumption.

5.10 $P(X=3|n=4,\pi=0.4) = 0.1536$ =BINOMDIST(3,4,0.4,0) where X is the number of Friday afternoons worked by senior partner.

5.12 a. $\pi=0.2$, $n=10$
$E(X) = 10(0.2) = 2$, $Var(X) = 10(0.2)(0.8) = 1.6$
Thus, the standard deviation is 1.265.

b. $\pi=0.5$, $n=17$
$E(X) = 17(0.5) = 8.5$, $Var(X) = 17(0.5)(0.5) = 4.25$
Thus, the standard deviation is 2.06.

c. $\pi=0.75$, $n=30$
$E(X) = 30(0.75) = 22.5$, $Var(X) = 30(0.75)(0.25) = 5.625$
Thus, the standard deviation is 2.37.

5.14 a. Let X be the number still use the exerciser five years later in a sample of 3 purchasers. $E(X) = 2.1$ says that 2.1 of these 3 purchasers, on average, will be using the exercisers over an infinite number of samples each of size 3.

b. $P(X=0|n=3,\pi=0.7) = 0.027$

5.16 $P(X \geq 7|n=10,\pi=0.5) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$ = 0.1719. This probability could be viewed as "large". Thus, it would not be unusual to find seven or more regular users when five(=10(0.5)) is the expected number for a sample of 10 and the observed seven could be considered to be "consistent" with the expected five on average. This implies that seven out of 10 can be observed when five is the true average. Thus, we might again question Nordic Track's ad.

5.18 From the given information, $\pi=0.25$(quarter of those who took part in Desert Storm were black) and n=291 where π is the probability of a black losing his/her life by chance.

a. The expected number of blacks losing their lives is $E(X) = 291(0.25) = 72.75$. But 53 blacks actually lost their lives. Thus, we can conclude that fewer blacks died than would be expected by chance.

b. $P(X \leq 53|n=291,\pi=0.25) = 0.003652$
=BINOMDIST(53,291,0.25,1)

Because n=291 is very large, we can use a normal approximation to binomial distribution, as introduced in Chapter 6, and provided here for instructor▪s use only. That is, the random variable X approximately follows a normal distribution with a mean of 72.75 and a standard deviation of 7.39($= \sqrt{291(0.25)(0.75)} = \sqrt{54.56}$). Thus,

$P(X \leq 53|n=291,\pi=0.25) = P(z \leq \frac{53-72.75}{7.39}) = P(z \leq -2.67)$
= 0.0038 where z is the standard normal random variable.

5.20 $P(X=2|n=4,\pi=0.9) = 0.049$ =BINOMDIST(2,4,0.9,0)
$P(X=2|n=4,\pi=0.5) = 0.375$ =BINOMDIST(2,4,0.5,0)

Because the probability of getting 2 or 4 beating the index with a flip of the coin is higher than the claimed 90 percent accuracy, the idea that the pros▪ ability is like a flip of the coin is supported.

5.22 a) 0.96059601 =BINOMDIST(0,4,0.01,0)
b) 0.03881196
c) 0.00058806
d) 0.00000396

e) 0.00000001 =BINOMDIST(4,4,0.01,0)

5.24 $P(X \geq 51 | n=53, \pi=0.5) = 0 \approx 1.584E-13$ =1-BINOMDIST(50,53,0.5,1) where X is the number predicting expansion.

5.26 a. $\mu = E(X) = 3(0.8) = 2.4$ where X is the number of error free items in three trials.

b. $E(X) = \Sigma x_1 p(x_1) + \Sigma x_2 p(x_2) + \Sigma x_3 p(x_3)$
= 1(0.8) + 0(0.2) ---- for the first trial
+ 1(0.8) + 0(0.2) ---- for the second trial
+ 1(0.8) + 0(0.2) ---- for the third trial
= 2.4
where X_1, X_2 and X_3 denote the number of error free items in the first, second and third trial, respectively.

c. The average number of error free items per sample, over infinite number of random samples of each size three, is 2.4.

d. $\sigma^2 = Var(X) = 3(0.8)(0.2) = 0.48$ where X is the number of error free items in three trials.

e. $Var(X) = \Sigma(x_1-\mu_1)^2 p(x_1) + \Sigma(x_2-\mu_2)^2 p(x_2) + \Sigma(x_3-\mu_3)^2 p(x_3)$
= $(1-0.8)^2(0.8) + (0-0.8)^2(0.2)$ -- for the first trial
+ $(1-0.8)^2(0.8) + (0-0.8)^2(0.2)$ -- for the second trial
+ $(1-0.8)^2(0.8) + (0-0.8)^2(0.2)$ -- for the third trial
= 0.16 + 0.16 + 0.16 = 0.48
where X_1, X_2, X_3 denote the number of error free items and μ_1, μ_2, μ_3 ($\mu_1=\mu_2=\mu_3=0.8$) denote the expected (mean) number of error free items in the first, second and third trials, respectively.

5.28 The probability of getting one totally error free item is $P(X=1|n=3,\pi=0.8) = 0.096$. The probability 0.096 is relatively "large." Thus, it is not be unusual to observe one error free item when 2.4 is the expected number. We can conclude that the assembly line is running in a way that the probability of any given item being error free is a 0.8.

5.30 $\pi=0.50$ and n=5 are given where π is the probability of getting a honest VCR repair center. Letting X be the number of honest VCR repair centers, the probability of getting 2 such centers would be $P(X=2|n=5,\pi=0.50) = 0.3125$.

5.32 If we use the binomial distribution with π=0.75(three quarters) and n=464 where π is the probability of an economist disagreeing with the proposition, then, letting X be the number of economists who disagree in a sample of 464,

P(X=343|n=464,π=0.75)   where 343 = (464)(0.739)

$$= C(464,343)(0.75)^{343}(0.25)^{121} = \frac{464!}{343!\,121!}(0.75)^{343}(0.25)^{121}$$

$$= 0.0366$$

Because the size of the pool from which the sample of 464 was drawn is not large, we cannot put much faith that the probability (of getting an economist who disagrees with the proposition) is fixed from trial to trial. The use of binomial is questionable. [The hypergeometric distribution is more appropriate but not covered in this book. It is contained in William E. Becker, *Statistics for Business and Economics* (South-Western, 1995).]

5.34 If only guessing is involved, the probability that a blindfolded woman would correctly identify his or her partner is π = 1/3. P(X$\geq$50|n=72,π=1/3) $\approx$ 0
= 4.23256E-10 =1-BINOMDIST(49,72,1/3,1)
This probability is too small to believe that they were just guessing in the identification of their partners.