Chapter 4

4.2 The median tells us that 50 percent of the nuns are 65 years or older and 50 percent are younger than 65 years. Thus, the probability of selecting a nun at random who is 65 years or older is approximately 0.5 and this probability can be regarded as fixed assuming there is a large selection pool of nuns.

4.4 The expected value of a random variable differs from an arithmetic average calculated with relative frequency data because the expected value is calculated with probabilities. Thus, the expected value is not based on what has occurred but on what would occur over an infinite number of trials.

4.6 Because the expected payoff($50) is less than the cost of the policy($210), in the long run you would be better off not buying it, but if you are risk averse you might still buy it.

4.8 If someone believes that the probability of TV failure is greater than 0.19192 then the policy is a good buy, but then the TV itself is a bad buy: why buy something that has a 20 percent chance of major failure? On the other hand, if someone was extremely risk averse and unable to insure themselves with a sufficiently large bank account to replace the TV set, then he or she might consider buying the service contract.

4.10 a. The random variable X is the profit to be made on sale of a 27-inch Schwinn bikes.

b. The expected value is $40.50 as shown in the following calculation; i.e., on average, the dealer will make $40.50 on the sale of a 27-inch model.

c. The variance is 62.25 square dollars:

| x | P(x) | xP(x) | x-E(X) | $(x-E(X))^2$ | $(x-E(X))^2P(x)$ |
|---|---|---|---|---|---|
| 35 | 0.6 | 21.0 | -5.5 | 30.25 | 18.150 |
| 45 | 0.3 | 13.5 | 4.5 | 20.25 | 6.075 |
| 60 | 0.1 | 6.0 | 19.5 | 380.25 | 38.025 |
| Sum | | 40.5 | | | 62.250 |

d. On 100 sales, expected profits are $4,050; E(100X) = 100E(X) = 4050.

4.12 a. The expected value is 3.00, as calculated below.

b. The variance is calculated to be 1.54, and the standard deviation is thus 1.24.

c. Within ±2 standard deviations around the mean {i.e., between 0.52[=3-2(1.24)] and 5.48[=3+2(1.24)]} are 96 percent(=0.09+0.23+0.32+0.23+0.09) of the distribution.

d. The result is somewhat consistent with the "rule of thumb," which states that there are about 95 percent of total items within ± 2 standard deviations around the mean because the given distribution is symmetric and approximately bell-shaped.

| x | P(x) | xP(x) | x-E(X) | $(x-E(X))^2$ | $(x-E(X))^2P(x)$ |
|---|---|---|---|---|---|
| 0 | 0.02 | 0.00 | -3 | 9 | 0.18 |
| 1 | 0.09 | 0.09 | -2 | 4 | 0.36 |
| 2 | 0.23 | 0.46 | -1 | 1 | 0.23 |
| 3 | 0.32 | 0.96 | 0 | 0 | 0.00 |
| 4 | 0.23 | 0.92 | 1 | 1 | 0.23 |
| 5 | 0.09 | 0.45 | 2 | 4 | 0.36 |
| 6 | 0.02 | 0.12 | 3 | 9 | 0.18 |
| Sum | | 3.00 | | | 1.54 |

4.14 A risk neutral person is only interested in the average net return over the long run; he or she does not care about risk. A risk neutral person would prefer investment C because it has the highest expected net return.

4.16 A risk neutral person prefers investment B because it has a higher expected net return.

4.18 a. Marginal probabilities are calculated as follows:

| | $A_1$ | $A_2$ | $A_3$ | marginal |
|---|---|---|---|---|
| $B_1$ | 0.12 | 0.03 | 0.10 | 0.25 |
| $B_2$ | 0.21 | 0.02 | 0.18 | 0.41 |
| $B_3$ | 0.11 | 0.09 | 0.14 | 0.34 |
| marginal | 0.44 | 0.14 | 0.42 | 1.00 |

b. $P(A_1 \text{ and } B_1) = 0.12$

c. $P(A_1 \text{ or } B_2) = P(A_1) + P(B_2) - P(A_1 \text{ and } B_2)$
$= 0.44 + 0.41 - 0.21 = 0.64$

d. $P(A_1|B_1) = P(A_1 \text{ and } B_1)/P(B_1) = 0.12/0.25 = 0.48$

e. $P(A_1 \text{ and } B_2) = 0.21 \neq P(A_1)\ P(B_2) = (0.44)(0.41) =$

0.1804 Thus, $A_1$ and $B_2$ are dependent.

4.20 If $P(A_1)$ is the probability of a taxpayer getting a field audit, and $P(A_2)$ is the probability of a taxpayer getting an office audit, and P(B) is the probability of a taxpayer getting audited(either a field audit or an office audit), then $P(A_1|B) = 0.2$, $P(A_2|B) = 0.8$ and $P(B) = 0.009$. The expected additional payment for a taxpayer who is notified that he or she is going to be audited is $5,038.6:

$E(X) = (15801)(0.2) + (2348)(0.8) = 5038.6$

4.22 Assuming the three investments are available at a price of $12,000, the portfolio investment, investment D, has 1/3 in each investment; i.e., D = (1/3)A + (1/3)B + (1/3)C. Thus, the expected value of D is E(D) = E[(1/3)A + (1/3)B +(1/3)C]
= (1/3)E(A) + (1/3)E(B) + (1/3)E(C)
= (1/3)(1000) + (1/3)(1500) + (1/3)(2000) = $1,500.

4.24 Because A, B and C are independent, the variance of D is Var(D) = Var[(1/3)A + (1/3)B + (1/3)C] = (1/9)Var(A) + (1/9)Var(B) + (1/9)Var(C)= $(1/9)(500^2) + (1/9)(700^2) + (1/9)(1150^2) = 229166.67$; the standard deviation is $478.71.

4.26 a. E(X) = 1(0.4) + 2(0.1) + 3(0.3) + 4(0.2) = 2.3

b. Because two probability distributions for X and Y are the same, E(Y) = E(X) = 2.3.

c. E(X-Y) = E(X) - E(Y) = 0

4.28 a. Var(X) = 1.41 as in the following calculation:

| x | P(x) | xP(x) | $(x-E(X))^2$ | $(x-E(X))^2P(x)$ |
|---|---|---|---|---|
| 1 | 0.4 | 0.4 | 1.69 | 0.676 |
| 2 | 0.1 | 0.2 | 0.09 | 0.009 |
| 3 | 0.3 | 0.9 | 0.49 | 0.147 |
| 4 | 0.2 | 0.8 | 2.89 | 0.578 |
| Sum | | 2.3 | | 1.410 |

b. Var(Y) = 1.41 as in the following calculation:

| y | P(y) | yP(y) | $(y-E(Y))^2$ | $(y-E(Y))^2P(y)$ |
|---|---|---|---|---|
| 4 | 0.2 | 0.8 | 2.89 | 0.578 |
| 3 | 0.3 | 0.9 | 0.49 | 0.147 |
| 2 | 0.1 | 0.2 | 0.09 | 0.009 |
| 1 | 0.4 | 0.4 | 1.69 | 0.676 |
| Sum | | 2.3 | | 1.410 |

c. The Cov(X,Y) = 0, as shown by the calculations in the table on the next page.

| | | **X = 1** | **X = 2** | **X = 3** | **X = 4** | **Sum** |
|---|---|---|---|---|---|---|
| **Y=4** | P(xy) | 0.08 | 0.02 | 0.06 | 0.04 | |
| | $(x-\mu_x)(y-\mu_y)$ | -2.21 | -0.51 | 1.19 | 2.89 | |
| | $(x-\mu_x)(y-\mu_y)P(xy)$ | -0.1768 | -0.0102 | 0.0714 | 0.1156 | 0 |
| **Y=3** | P(xy) | 0.12 | 0.03 | 0.09 | 0.06 | |
| | $(x-\mu_x)(y-\mu_y)$ | -0.91 | -0.21 | 0.49 | 1.19 | |
| | $(x-\mu_x)(y-\mu_y)P(xy)$ | -0.1092 | -0.0063 | 0.0441 | 0.0714 | 0 |
| **Y=2** | P(xy) | 0.04 | 0.01 | 0.03 | 0.02 | |
| | $(x-\mu_x)(y-\mu_y)$ | 0.39 | 0.09 | -0.21 | -0.51 | |
| | $(x-\mu_x)(y-\mu_y)P(xy)$ | 0.0156 | 0.0009 | -0.0063 | -0.0102 | 0 |
| **Y=1** | P(xy) | 0.16 | 0.04 | 0.12 | 0.08 | |
| | $(x-\mu_x)(y-\mu_y)$ | 1.69 | 0.39 | -0.91 | -2.21 | |
| | $(x-\mu_x)(y-\mu_y)P(xy)$ | 0.2704 | 0.0156 | -0.1092 | -0.1768 | 0 |
| | | | | | **Grand Sum** | 0 |

4.30 The expected win is E(X) = 10(1/6) + (-10)(5/6) = -6.67($) because you could win $10 with a probability of 1/6 and you could lose $10 with a probability of 5/6. On average you will lose $6.67 per play.

4.32 What is important to the insurance company is not only the probability of a pizza delivery driver having an accident per mile driven, but also is the amount of claim per accident. If the amount of claim is large, the expected loss could also be large, although the probability of accident is not high.

4.34 The word "expects" implies the expected value of Pepsi's market share. It is a mean market share based on perceived probabilities of success and failure.

4.36 Similar to the economist=s claim that state lotteries are Aa tax on stupid people@ the April 23, 1995, issue of *The Sunday Times* (of London, England) featured an article in which state run lotteries were labeled Athe dumb tax.@ These negative labels are associated with the fact that the expected payoff for a lottery ticket is less than the cost to buy it. Thus, the difference between the cost of a state lottery ticket and its expected payoff is a tax levied on those who willing buy them.

4.38 a. They failed to deliver the expected profits because of the increase in cost of administrating lotteries.

b. If the lottery profits had been placed into more effective education, there would have been more educated people and thus an increase in numbers of those educated people who are not willing to buy lottery tickets might have led to future reduction in lottery revenues.

4.40 a. The twin has the greatest chance of going down because P(1st or 2nd) = P(1sr) + p(2nd) -P(1st or 2nd)
= 0.001 + 0.001 - (0.001)(0.001)= 0.001999

b. Assuming engine failure is independent.

c. The break-even point for an insurance policy on the single engine plane is $1,000. The break-even point for an insurance policy on the twin engine plane is $3,998, as calculated below:

| | | | | | |
|---|---|---|---|---|---|
| 0.001 | -1,000,000 | -1000 | 0.001999 | -2,000,000 | -3998 |
| 0.999 | 0 | 0 | 0.998001 | 0 | 0 |
| | | -1000 | | | -3998 |

4.42 a. Investment B has the highest expected net return; thus, a risk neutral investor will select it.

b. Var(new investment) = $(0.25)(9*10^6)$ + $(0.25(16*10^6)$
St.Dev.(new investment) = $0.5(5*10^3)$ = $2,500

4.44 If the wheels had no losing green slots then the escalating strategy would work, but the wheels have either one or two losing green slots which implies that even in the long run this strategy need not produce a break-even position for a player. Furthermore, this strategy could require the player to be placing bets that far exceed his or her assets, which no casino will tolerate for long.

4.46 The break even insurance premium is $1,800.

| | | | |
|---|---|---|---|
| 0.01 | 0.05 | 0.94 | |
| 80000 | 20000 | 0 | |
| 800 | 1000 | 0 | 1800 |

4.48 Let return C = (**2**)A + (**2**)B, where A and B are returns from two other investments of a fixed amount (e.g., one can put $1,000 into the investment with return A, or $1,000 into the investment with return B, or $500 into each for return C). Then, the standard deviation of C is the square root of

Var[(**2**)A+(**2**)B] = (1/4)Var(A)+(1/4)Var(B)+2(**2**)(**2**)Cov(A,B). For Cov(A,B) $\leq$ 0 the standard deviation of C can be far less than the standard deviations of B or A, as seen in the answer to Exercise 4.25, for example.