Chapter 10

10.2 H_0: $\mu_2 - \mu_1 = 0$ vs. H_A: $\mu_2 - \mu_1 \neq 0$, where μ_1 is the mean gas mileage of domestic cars and μ_2 is the mean gas mileage of imported cars.

$\bar{x}_1 = 34.3$, $\bar{x}_2 = 34.8$, $\sigma^2_1 = 2.1$ and $\sigma^2_2 = 1.9$ are given. Thus,

$$z = \frac{(\bar{x}_2 - \bar{x}_1) - D_0}{\sqrt{\frac{\sigma^2_1}{n_1} + \frac{\sigma^2_2}{n_2}}} = \frac{(34.8 - 34.3) - 0}{\sqrt{\frac{2.1}{18} + \frac{1.9}{14}}} = 0.995$$

The calculated $z = 0.995$ is smaller than the critical $z_{0.025} = 1.96$. Thus, H_0 cannot be rejected.

10.4 a. H_0: $\mu_2 - \mu_1 = 0$ vs. H_A: $\mu_2 - \mu_1 \neq 0$, where μ_1 is the mean reading time of older people and μ_2 is the mean reading time of younger people. With $\bar{x}_1 = 0.12$, $\bar{x}_2 = 0.11$, $\sigma^2_1 = 0.015^2$ and $\sigma^2_2 = 0.011^2$, the test statistic is

$$z = \frac{(\bar{x}_2 - \bar{x}_1) - D_0}{\sqrt{\frac{\sigma^2_1}{n_1} + \frac{\sigma^2_2}{n_2}}} = \frac{(0.11 - 0.12) - 0}{\sqrt{\frac{0.015^2}{16} + \frac{0.011^2}{12}}} = -2.04$$

The calculated $z = -2.04$ is beyond the critical $-z_{0.025} = -1.96$. Thus, H_0 is rejected at a 0.05 Type I error level. There is still a difference in mean reading times between the older and the younger people.

b. Along with the assumption of independent samples, we need to assume that the two populations are normal so that the sampling distribution of $(\bar{x}_2 - \bar{x}_1)$ can be treated as normal because two sample sizes are not large enough to invoke the Central Limit Theorem.

10.6 H_0: $\mu_2 - \mu_1 \geq 0$ vs. H_A: $\mu_2 - \mu_1 < 0$, where μ_1 is the mean income($) of academic engineers and μ_2 is the mean income($) of industry engineers.

$\bar{x}_1$ = 95,000, $\bar{x}_2$ = 84,000, $s^2_1 = 9{,}000^2$ and $s^2_2 = 7{,}000^2$ are given. Provided that two population variances are the same and two samples are independently drawn from normal populations, the test statistic is calculated to be

$$t = \frac{(\overline{x_2} - \overline{x_1}) - D_0}{\sqrt{\frac{s^2_1(n_1-1)+s^2_2(n_2-1)}{n_1+n_2-2}\;\frac{n_1+n_2}{n_1 n_2}}}$$

$$= \frac{(84000-95000)-0}{\sqrt{\frac{(9000^2)(11)+(7000^2)(12)}{23}\;\frac{12+13}{(12)(13)}}} = -3.43$$

The calculated t = -3.43 is beyond the critical value of $-t_{(0.05,df=23)}$ = -1.714. Thus, H_0 is rejected in favor of H_A at the 0.05 Type I error level. We can conclude that academic engineers have higher mean income than their industry counterparts.