The Toy Train Problem

At Christmas time last year, the Indianapolis Children's Museum made a large display using a toy train, gingerbread houses, plaster mountains, and other small toys. The result looked something like the map that's shown on the back of this page.

The display covered a very large table; and, it had three mountains that were made with paper and plaster. The tallest mountain was slightly more than two meters tall. That's more than 200 centimeters – or more than six feet! A small gingerbread house called "Santa's Workshop" was at the top of this mountain – 210 centimeters. The X on the map shows exactly where Santa's Workshop was located.

Tower Mountain was smaller than Santa's mountain. But, it was still more than 190 centimeters at its peak. … The top of the big rock candy mountain was at 180 centimeters.

Unfortunately, the grade of the train track was too steep last year. So, the toy train wasn't powerful enough to climb the mountain. … This year, the people who are building the toy train display hope to solve this problem in three ways. First, they are going to use a toy version of the famous train, Rueben Wells. It is very powerful and can go up and down a 10% grade. Second, they are planning to build a bridge between two of the mountains. And third, they are going to plan the tracks very carefully to make sure that the grade of the tracks is never greater than 10%. To do this, they are guessing that they might need to spread the three mountains a bit farther apart from one another - and to add more material to the mountains to make them wider. But, they're not sure if they'll really need to do this.

Two facts ARE certain. First, the museum director insists that Santa's mountain must be at least 200 centimeters tall. Second, the mountains will have to be made before the display can be assembled – and before the train can be tested. So, it's important to have a plan that should work for the path of the train.

<u>Your Task</u>:

1. Using the map on the back of this page, draw a path for the toy train. The path must be a closed loop that makes one stop at Santa's Workshop and another stop at the toy village. But, it can't be a simple circle. It must wind around the mountains; and, most of all, the grade of the tracks must be as small as possible – without wasting too much track. Remember, you can use one bridge that you can put between any two of the mountains.
2. Using your first path as a guide, draw a second path that you also think might work for the Christmas display. But, make this second path as short as possible by eliminating sections of track that are not needed to keep the grade small.
3. Write a letter to the people at the Indiana Children's Museum describing exactly how wide and deep the display will need to be so that the path you have drawn will never have a grade that is more than a 10% at it steepest point? (note: Describe how you figure out your answer to this question so that, if some other path is chosen for the train, the people who build the display will be certain that the grade of the tracks will never be greater than 10%.)

Notes about the Map

This map is a topographical map like the one that's described in the warm-up activity for this problem. The map's contour lines show the height of the mountains at the start of each level. Notice that the width and length of the map are not shown on the map. This is because they have not yet been determined. They will need to be big enough so that the mountains can be the heights that are shown – and so that the path of the train will never have more than a 10% grade. … One member of your team will have information on reading topographic maps while another will have information on grades (slope).

Notes about the Toy Train and Tracks

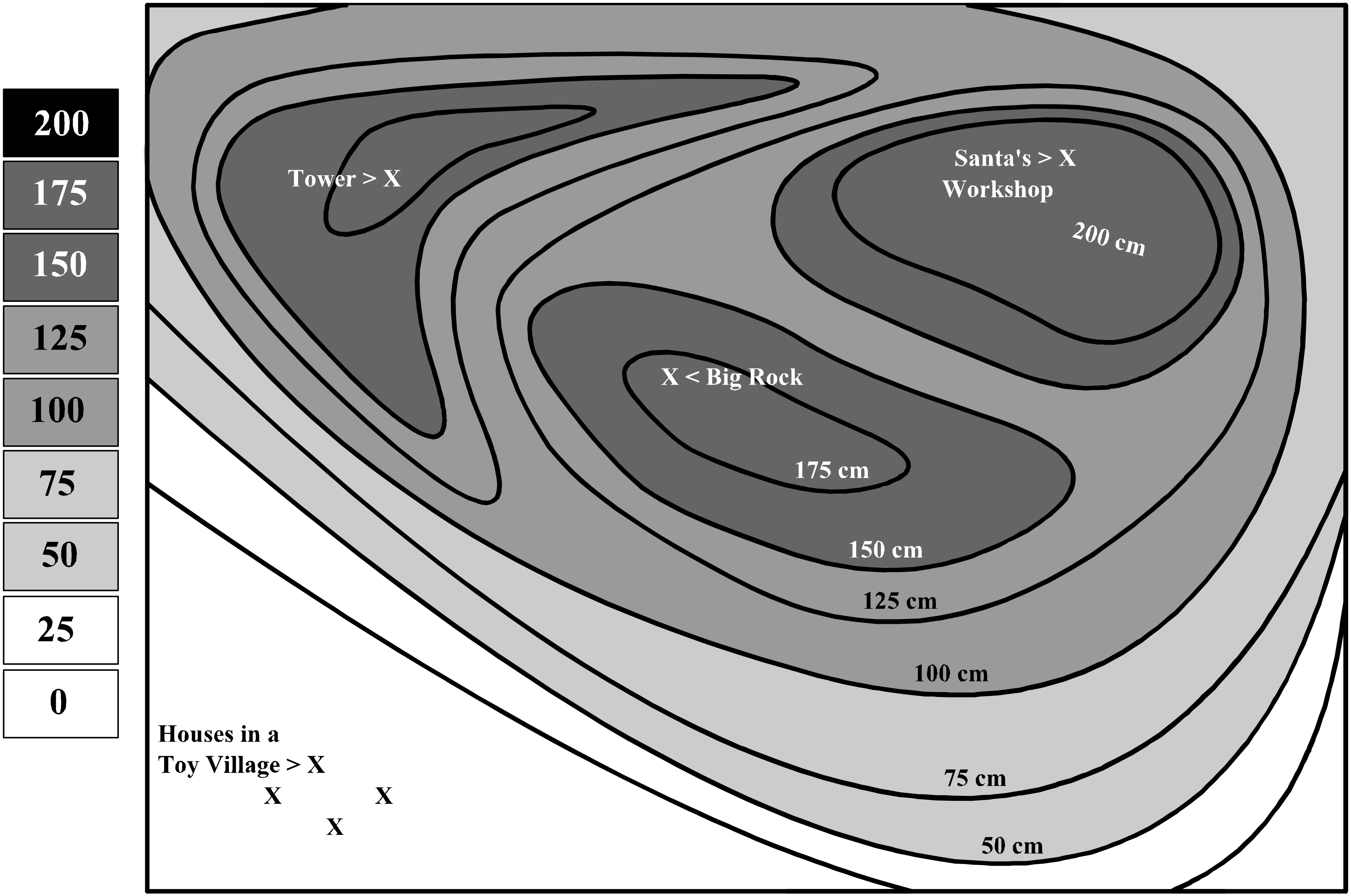
One member of your team will be given information about the kind of toy train track pieces are available. The size and shape of these pieces may limit what kind of paths you can make.

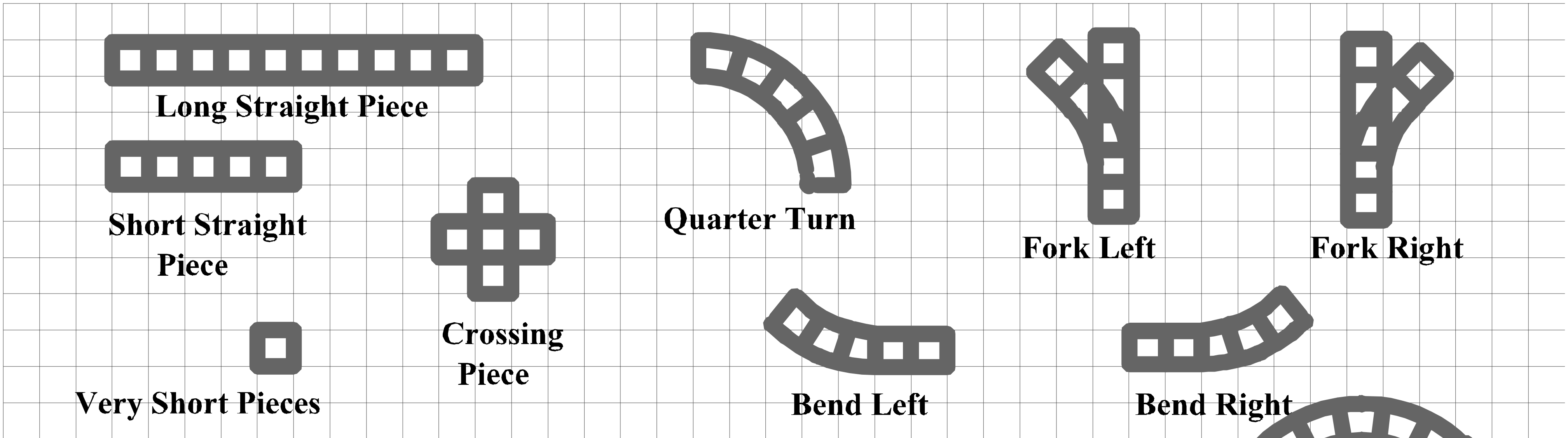
Warm-up Teams

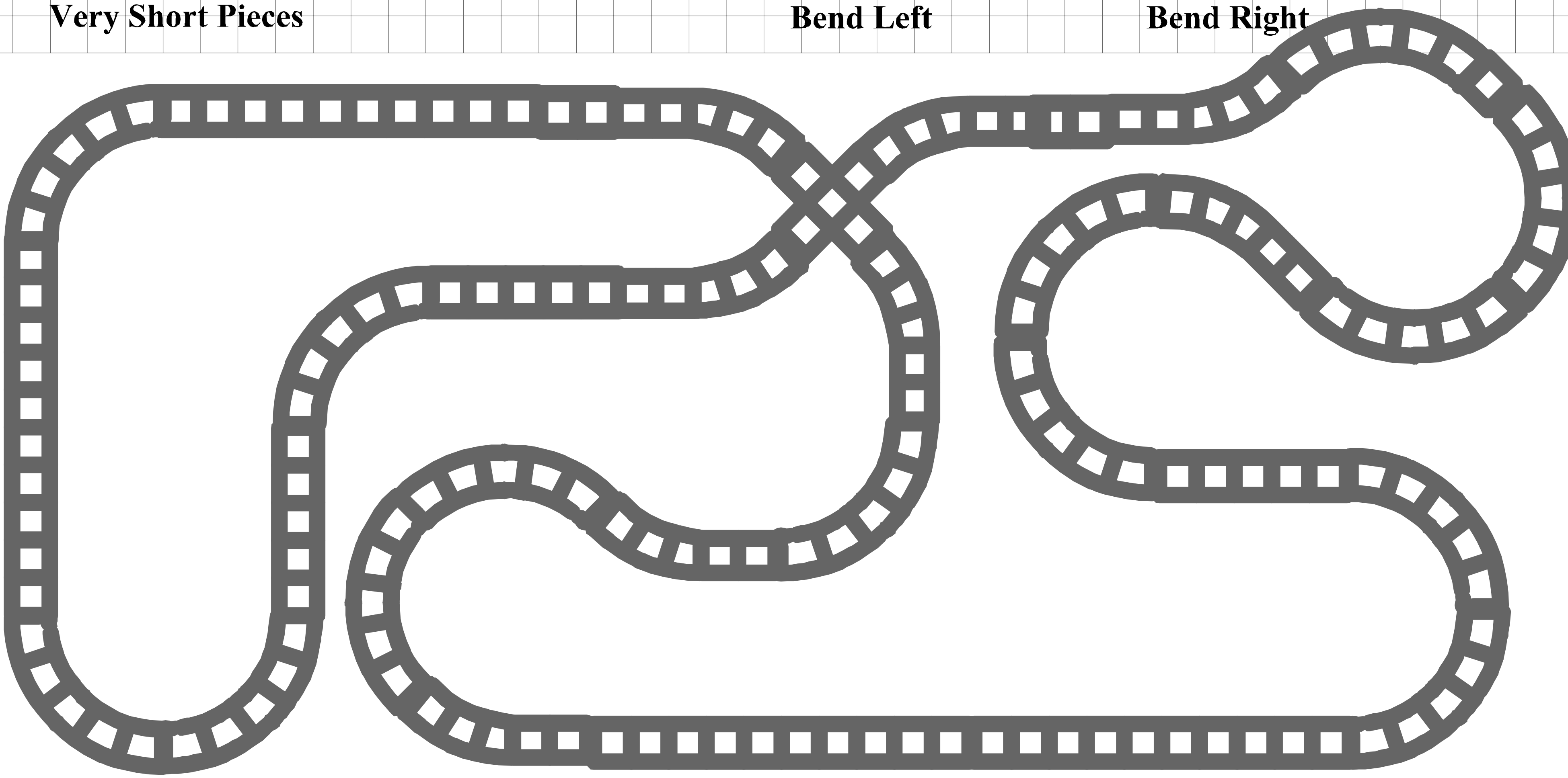
When students work on the Toy Train Problem, they'll work in three-person teams – with one expert drawn from each of the following sub-groups. Therefore, to get students ready to work on problem, divide the class into three sub-groups. Each sub-group will learn to be experts about one topic that's relevant to the Toy Train Problem. The three topics are:

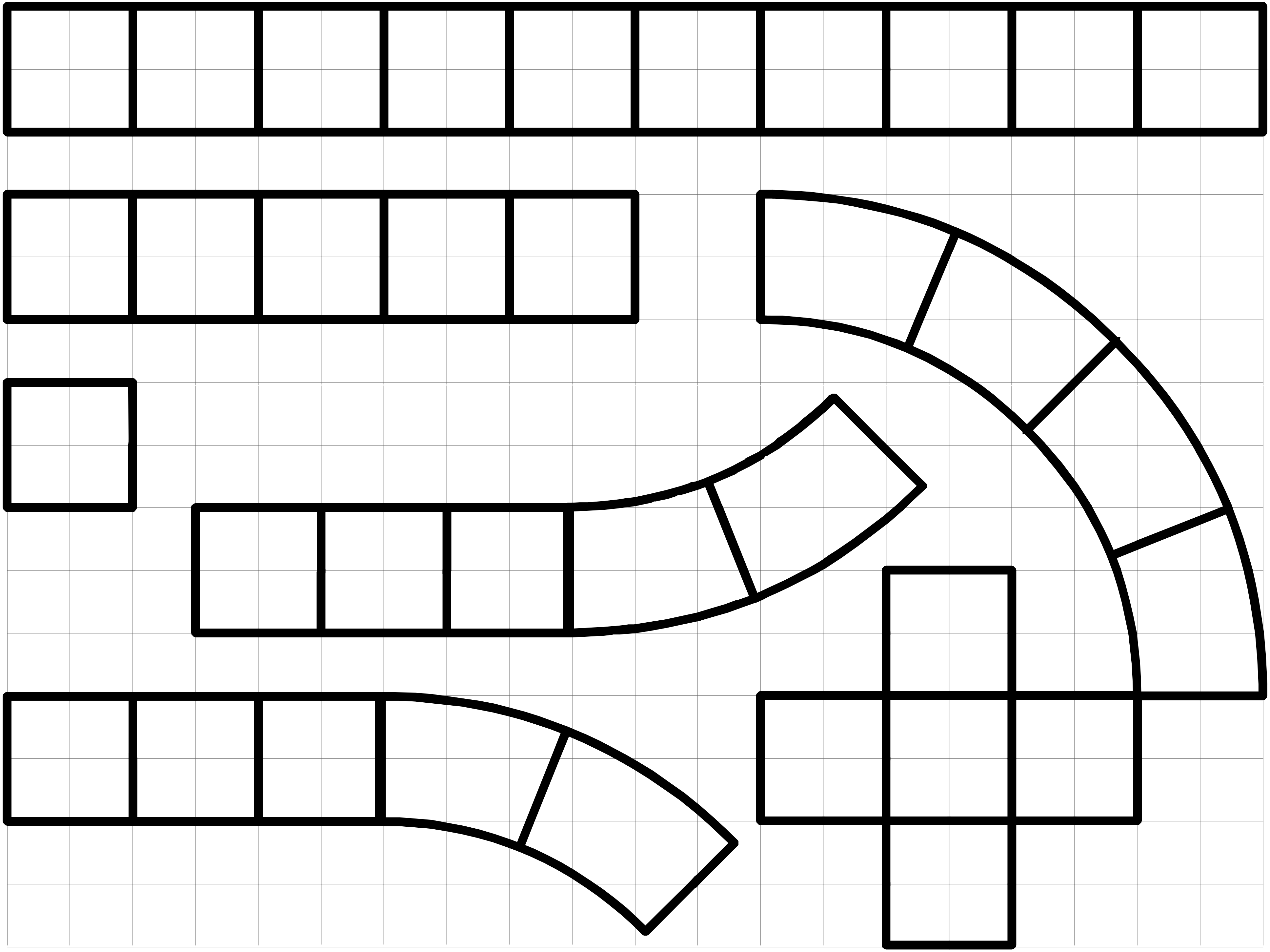
- Expert A: Learn what kind of paths can and cannot be made using specific pieces of track for a toy train.
- Expert B: Learn how to make topographic maps using clay disks.
- Expert C: Learn about the RUEBEN WELLS TRAIN and the idea of GRADE (i.e., slope).

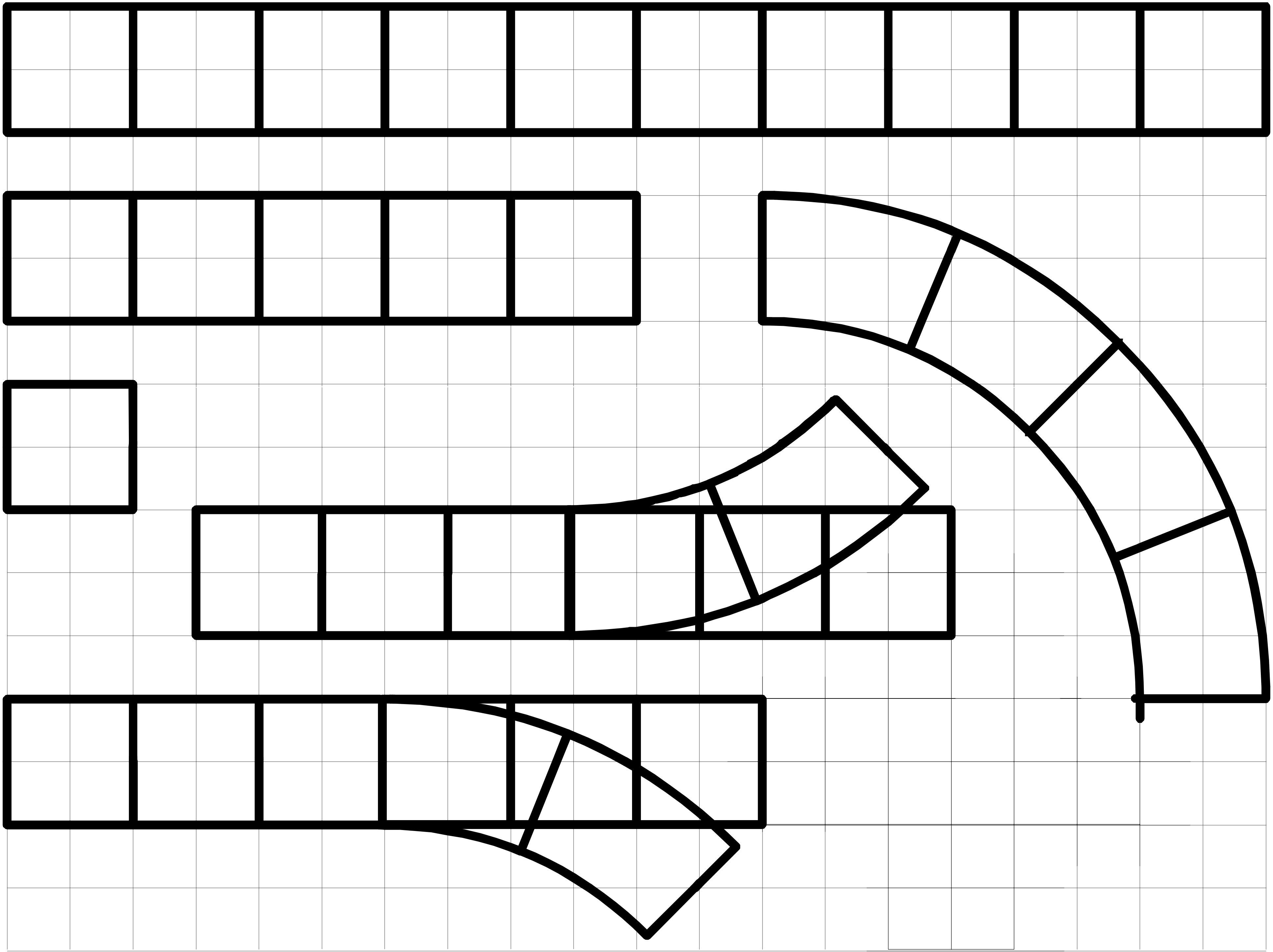
Toy Train Problem

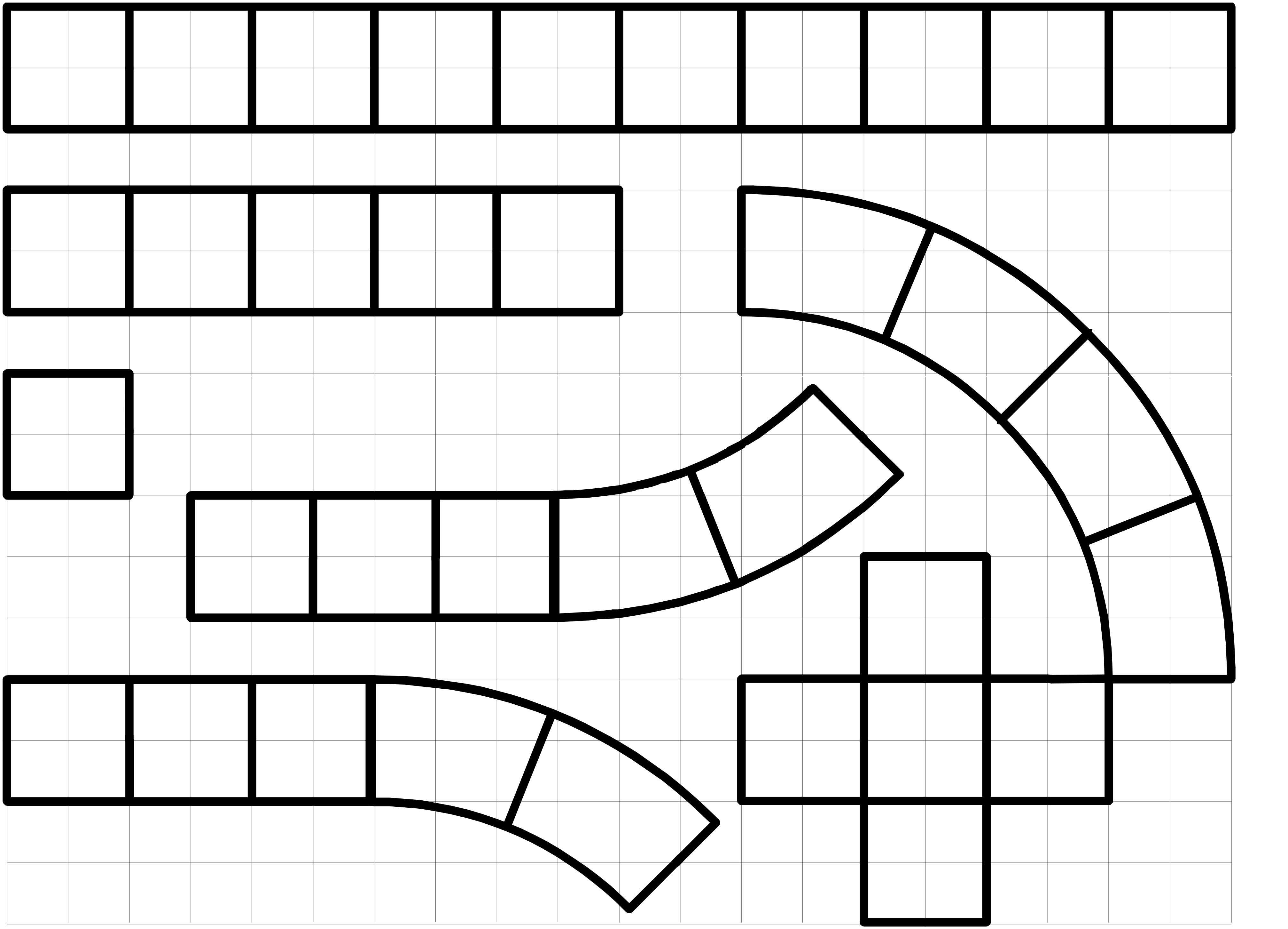



Long Straight Piece
Short Straight Piece
Very Short Pieces
Crossing Piece
Quarter Turn
Bend Left
Fork Left
Fork Right
Bend Right



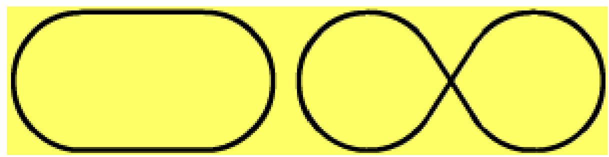




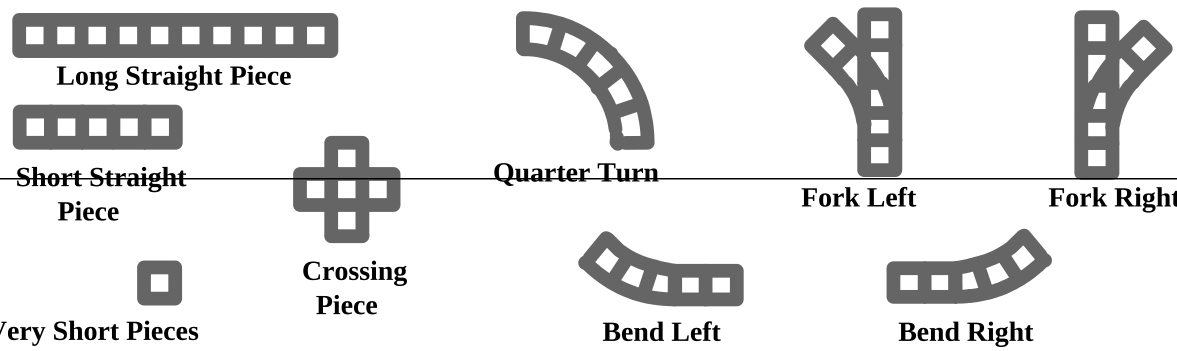


Expert A
Assembling Toy Train Tracks

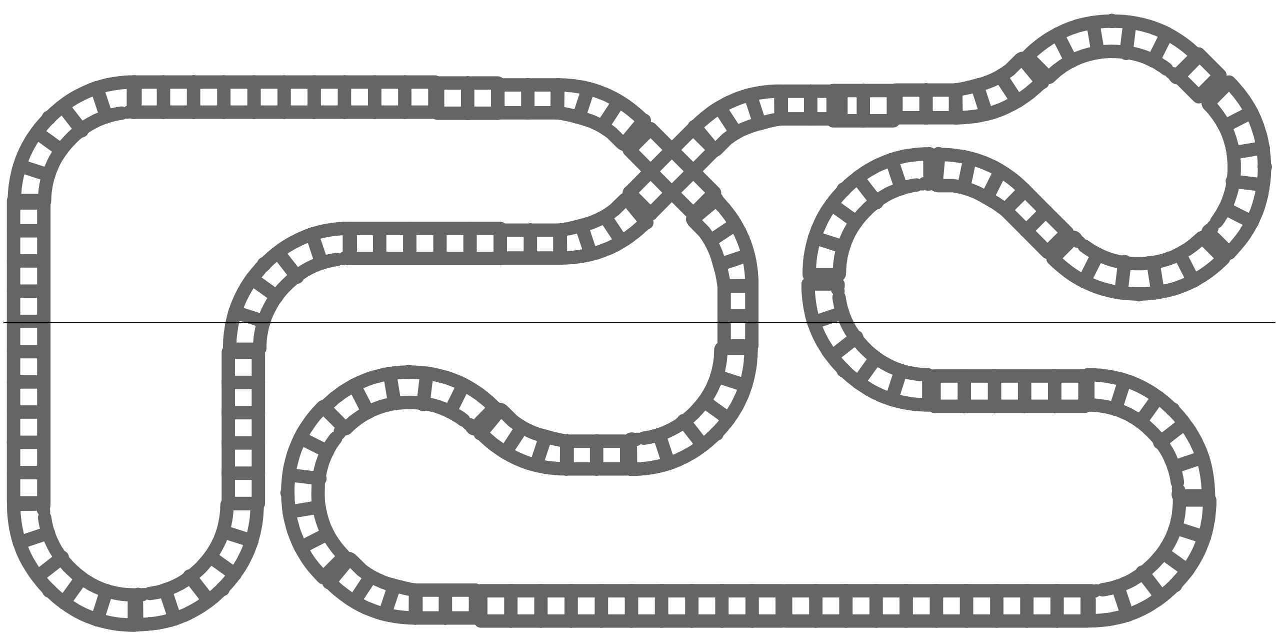
Chances are that someone in your group already knows some things about electric trains. Ask them to explain why electric train tracks usually are closed loops – like those shown below? If not, visit the Internet to learn more about the basics of model trains (http://www.nmra.org/beginner/basicplan.html is a good place to start).



On the back on this page, pictures are given of the toy train track pieces shown below. When you link several pieces together, they are flexible enough to bend slightly. But, even if some bending is allowed, it is not possible to make many kinds of paths.

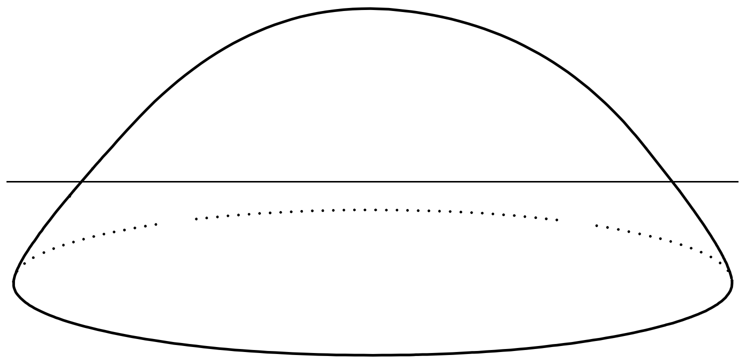


The picture below shows one path that is possible to make using as many as possible of the pieces given on the last few pages of this document. Cut the pieces and use them to make several other possible paths. Also, sketch several paths that are impossible to make with these pieces.

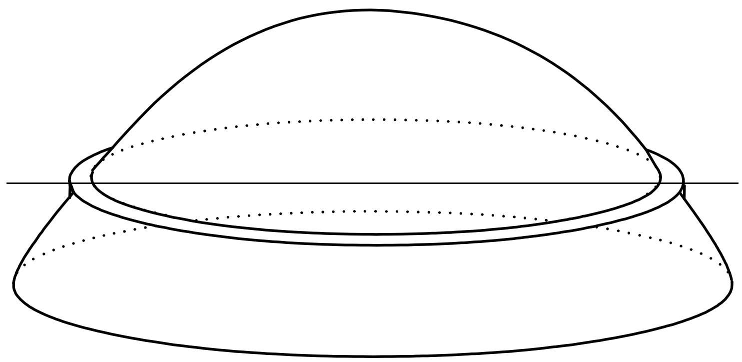


Making & Modifying Toy Mountains

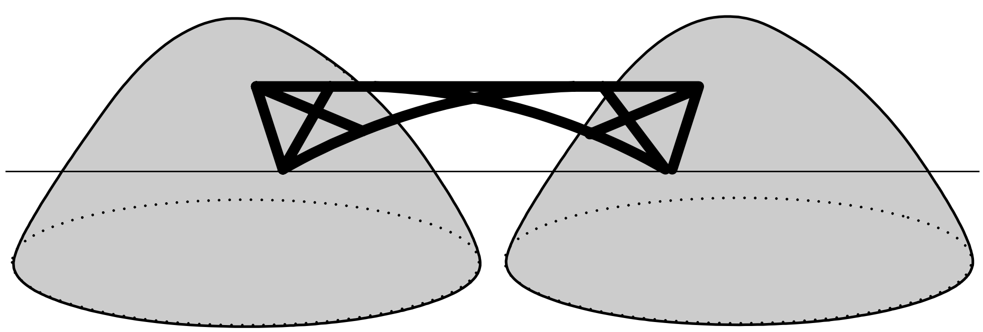
Probably, someone in your group already knows some things about making things with paper mache. Ask them to explain how to make a “mountain” for a toy train display. If not, visit the Internet to find out the basics of creating paper mache crafts (http://familycrafts.about.com/od/papermache/ is a good place to start). Discuss how you might be able to make a mountain that is more than a meter tall and more than two meters wide.



If you want to make a toy train track go around a mountain that you have built, you might need to modify the mountain slightly by making a ledge which makes a flat path for the train – similar to the one shown in the picture below. Discuss how you might be able to make such a ledge as a final step after the main parts of the mountain have dried and have become solid.

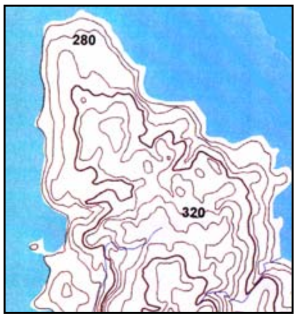


If you make more than a single mountain for a train layout, you might want to use balsa wood to build a bridge between mountains. The bridge might look something like the one shown below. Discuss how you might be able to make such a bridge for a toy train to cross.

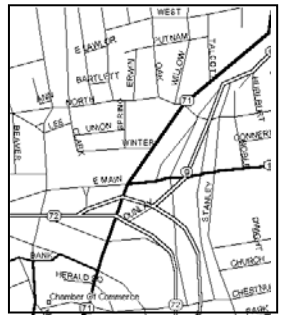


Fairview Students Explore Maps

What is a topographic map? This is a question that Fairview Elementary students recently considered. What makes a topographic map different from a regular street map? Here are examples of each type of map showing different places:



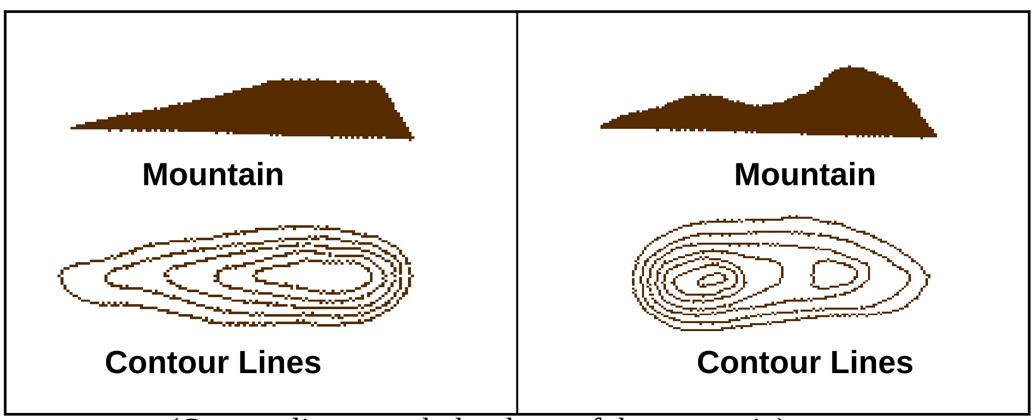
Topographic Map



Street Map

Fairview student Kim Wilson explains, "Topographic maps are different from regular street maps because they show how the land is shaped. By looking at a topographic map, you can tell where there are hills and valleys, and where the land is flat."

Topographic maps have special lines called "contour lines." A contour line is a line that joins points with the same elevation. If we think of the land as a layered cake, each line would be the beginning of a different layer. When contour lines are close together, there is not much space between the two layers indicating that the land is very steep. When the contour lines are far apart, there is more room between the layers, indicating the land is flat or is made up of gently rolling slopes.



(Contour lines match the shape of the mountain)

The Fairview Elementary class used their knowledge of contour lines and topographic maps to create their own mountains out of clay. Tom Roberts, a student in the class, stated, "We started with a pattern of the mountain that had contour lines for each layer. We used the pattern to cut out our layers of clay and stacked them up like a layered cake. To make it look more like a mountain, we filled in the areas between the layers. After we did that, we were able to tell how steep the land was."

Donna Mason, Fairview teacher, explained, "Our class is creating a model of the area around our school. They all knew how to read street maps, but didn't know what a topographic map was. Now they can make their model a little more realistic by adding the shape of the land."

Mason's class followed these steps to build their mountains:

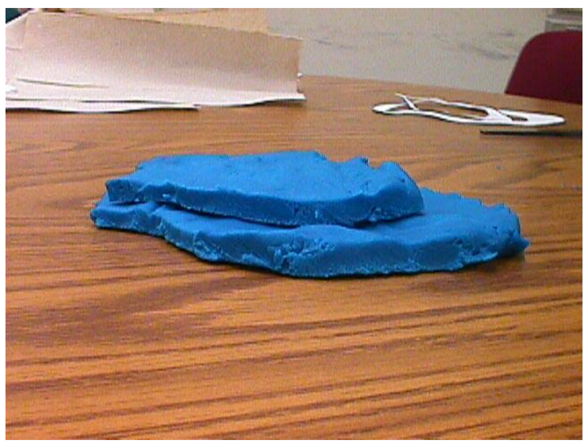
How to Create a Clay Mountain:

1. Roll out the clay into a flat circle (like a pizza) with a thickness of 1 cm.

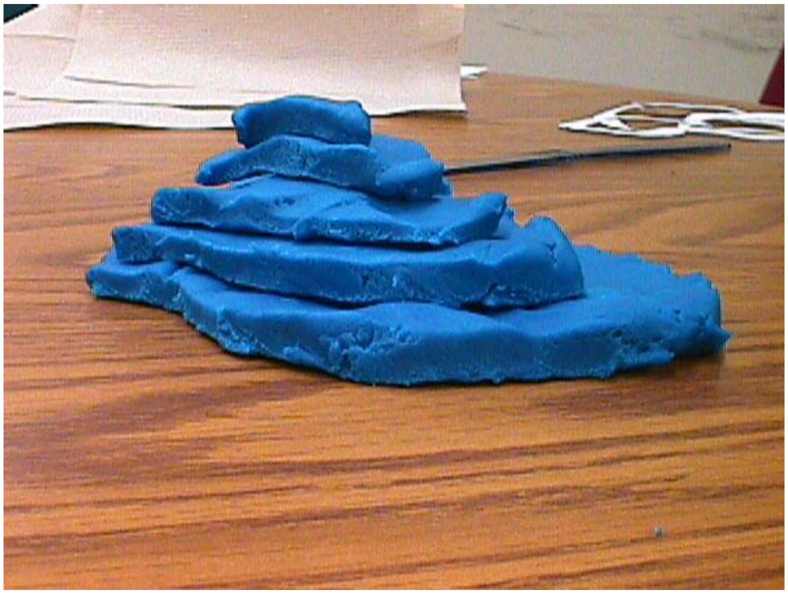
2. Take out the pattern of the mountain and cut around each contour line – you should have a cut-out (that resembles a doughnut) for each contour line.

3. Place the cut-outs on top of the circle of clay and trim around the outside of the pattern with a toothpick.

4. Stack up the clay cut-outs to resemble the original contour lines.

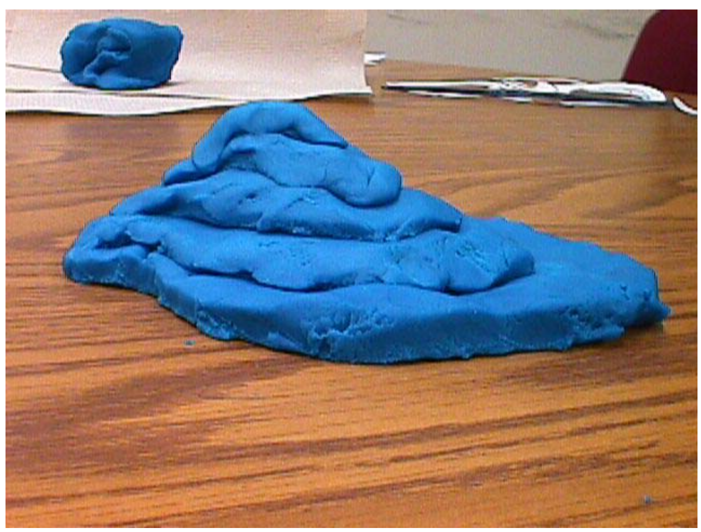


5. Your mountain should now resemble a staircase.



6. Place the extra clay between the layers of the mountain (stair steps) to make it look real.

7. You have built a clay mountain!



Readiness questions:

Answer the following questions before beginning the activity.

1. What does a topographic map show?

2. What is a contour line?

3. What do contour lines that are far apart mean? What do contour lines that are close together mean?

4. How did Donna Mason's class learn about topographic maps?

Activity:

Create a topographic (contour) map of the area around Fairview School. Use the information given Use the pictures of the clay mountain that Ms. Mason's class created.

Follow-up Questions:

Once you've completed your map, compare the mountain made by Ms. Mason's class to the topographic map you made.

1. What did the mountain look like where the lines on the map were close together?

2. What did the mountain look like where the lines on the map were far apart?

3. If you had to climb this mountain, which way would you go?

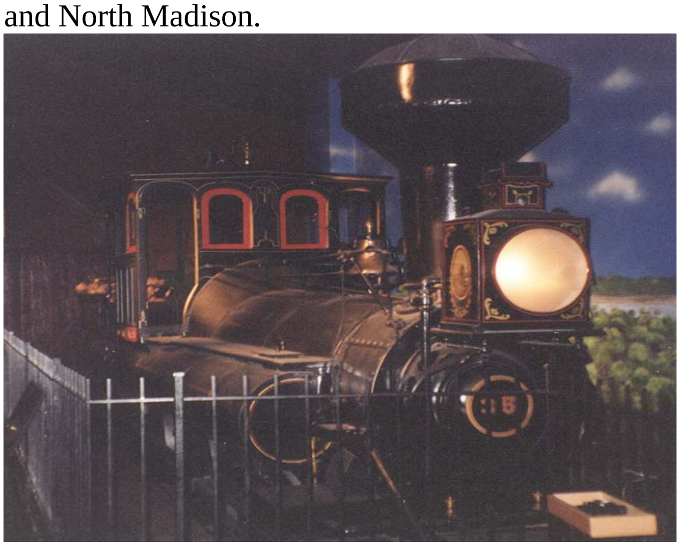
Expert C

INDIANAPOLIS CHILDREN'S MUSEUM HOUSES ONE OF INDIANA'S LARGEST RESIDENTS

INDIANAPOLIS –The Reuben Wells, once called the world's largest and most powerful locomotive, now calls The Children's Museum home. It weighs more than 112,000 pounds and is 36 feet long. This locomotive was created to climb a steep incline between the towns of Madison and North Madison, Indiana.

In the late 1830s, construction of the Madison and Indianapolis Railroad began. Engineers were soon faced with the challenge of connecting the Ohio River port of Madison with North Madison. From there the train could transport people and goods to Indianapolis and other locations. Between the two towns, the train faced a steep incline rising 311 feet per mile up 413-foot high limestone bluffs. This incline was so steep that a regular train would slip backwards while trying to move uphill. Thus, passengers and cargo from the Madison and Indianapolis Railroad had to be placed on small carts pulled by mules and horses to be carried up the hill.

Using these small carts proved to be expensive and slow. Men had to drive the mules or horses and transfer the cargo to and from the carts to the train. Thus, in 1868, Andrew Carthcart and Reuben Wells specially designed a locomotive they named the Reuben Wells to make the trip up the steep bluff. The locomotive was powerful and heavy enough to climb the hill between Madison and North Madison.



The Reuben Wells at the Indianapolis Children's Museum

Photo by Matthew Oates

Unlike other locomotives, the Reuben Wells pushed the railroad cars up the hill. Pushing rather than pulling the cars prevented the cars' couplings from breaking. The Reuben Wells usually ascended the hill in thirteen minutes at an average speed of six miles per hour. Cargo and passengers no longer had to be removed from the train and placed on carts!

Model at the Indianapolis Children's Museum showing how the Reuben Wells pushed the railroad cars up the hill.

Photo by Matthew Oates

The Reuben Wells remained in service until 1898. After 1898, various institutions such as Purdue University and the Pennsylvania Railroad stored the locomotive. In the late 1960s, Thomas M. Billings, the chairman of The Children's Museum Advisory Committee, learned that the locomotive was sitting in storage. He began petitioning to move the Reuben Wells to the museum in Indianapolis. On June 30, 1968 it was officially entrusted to The Children's Museum.

The Reuben Wells is remembered for its ability to transport goods and passengers up a 5.98% grade. Engineers refer to the 5.98% grade as a railroad gradient. Railroad gradients are expressed in percentages. A rise of one vertical foot for every 100 horizontal feet is called a 1% grade. Thus, a 5.98% grade represents a rise of 5.98 feet for every 100 horizontal feet. The steepest gradient currently in use on a main-line railroad in the United States is a 4.7% grade.

Expert C

REUBEN WELLS READINESS QUESTIONS

After reading the article, answer the following questions.

1). How were cargo and people originally transported from Madison to North Madison?
2). Why was the Reuben Wells designed?
3). How does the Reuben Wells differ from a traditional locomotive?
4) The picture below shows a slope with a 10% grade. This means that you go up 1 step for every 10 steps you move horizontally. Or, you go up 10 steps for every 100 steps you move horizontally. … Draw a picture of a 20% grade. Then, draw another picture of a 30% grade.
4). If you climbed a 6.5% grade, how much higher would you be after traveling 100 feet?
5). If a train rises 2.5 meters after traveling 100 meters, what percent grade did it climb?
6). What would be the percent grade of a rise of 6 feet for every 200 feet?

This Slope has a 10% Grade