

■ A Pipeline Flow Problem

The Setting. The Alaska pipeline consists of a network of pipes with various flow capacities (gallons/second). Flow through the network must satisfy three rules:

1. Flow is allowed only in the direction indicated on each pipe.
2. The actual flow in any pipe cannot exceed the pipe's capacity.
3. The amount of flow arriving at a node must equal the amount of flow leaving the node.

The Task. Given the graph theory model of the pipeline shown in Figure A.3 (the edge weights represent flow capacities, and the arrows indicate direction of flow), we are interested in the maximum possible flow from Prudhoe Bay to Seward.

1. Find the maximum possible flow. Provide a proof that your solution is indeed the largest possible flow and/or a general algorithm (method) for finding the maximum flow in a network.
2. If the maximum flow had to be increased by at least 3 units of flow, but for as little cost as possible, what changes would you propose? (Make some reasonable assumptions about relative costs.)

