

Suggested Answers for WarmUps for Lesson 19 Part II

A sample of 25 deliveries from an automobile parts supplier were timed and recorded. The sample revealed a mean of 57.79 minutes, and a standard deviation of 6.58 minutes. Delivery times are thought to be normally distributed. Use this information to answer the following THREE questions.

1.	Estimate a 96% confidence interval for the mean delivery time of all automobile parts from this supplier.
Answer	$57.79 \pm TINV(0.04,24) * \frac{6.58}{\sqrt{25}} = 57.79 \pm 2.857753 \Rightarrow [54.93, 60.65]$
2.	Re-estimate the 96% confidence interval for the mean delivery time of all automobile parts from this supplier, assuming n=100.
Answer	$57.79 \pm TINV(0.04,99) * \frac{6.58}{\sqrt{100}} = 57.79 \pm 1.369404 \Rightarrow [56.42, 59.16]$
3.	How does Question 1 differ from Question 2? Why?
Answer	Question 2 has a larger sample than question one, thus the standard error is smaller and hence the interval is narrower. A larger sample has more information, hence more certainty, hence a better, that is, narrower, interval estimate.
In 1991 the Gallup Poll asked a random group of 723 Americans if they had ever been bitten by a dog. Forty-two percent indicated that they had been bitten. Use this information to answer the following THREE questions.	
4.	Estimate a 98% confidence interval for the true proportion of Americans who have been bitten by a dog.
Answer	<p>Test for symmetry: $n*\pi=0.42*723=303.66$ and $n*(1-\pi)=0.58*723=419.34$, both greater than 5, thus may use the normal to estimate the distribution of the sample proportion.</p> $p \pm Z_{\frac{\alpha}{2}} * \sqrt{\frac{\pi * (1 - \pi)}{n}} = 0.42 \pm NORMSINV(0.99) * \sqrt{\frac{0.42 * (1 - 0.42)}{723}}$ $= 0.42 \pm 0.042702 \Rightarrow [0.38, 0.46]$ <p>OR Upper Bound : $NORMINV(0.99,0.42,0.018356) = 0.46$ Lower Bound : $= NORMINV(0.01,0.42,0.018356) = 0.38$</p>
5.	Estimate a 98% confidence interval for the true proportion of Americans who have been bitten by a dog assuming that the sample proportion was 10%.
Answer	Test for symmetry: $n*\pi=0.10*723=72.3$ and $n*(1-\pi)=0.90*723=650.7$, both greater than 5, thus may use the normal to estimate the distribution of the sample proportion.

	$p \pm Z_{\frac{\alpha}{2}} * \sqrt{\frac{\pi * (1 - \pi)}{n}} = 0.10 \pm \text{NORMSINV}(0.99) * \sqrt{\frac{0.10 * (1 - 0.10)}{723}}$ $= 0.10 \pm 0.03 \Rightarrow [0.07, 0.13]$ <p>OR Upper Bound : $\text{NORMINV}(0.99, 0.10, 0.011) = 0.13$ Lower Bound : $= \text{NORMINV}(0.01, 0.10, 0.011) = 0.07$</p>
6.	How does Question 4 differ from Question 5? Why?
Answer	Changing the sample proportion altered the place on the number line the interval would be centered, moving it from 0.42 to 0.10. This also changed the magnitude of the upper and lower bound. However, more importantly, this change affected the value of the margin of error because it affected the value of the standard error. The standard error when $\pi=0.42$ was 0.018356. When $\pi=0.10$, it was 3. The interval around the point estimate 0.42 is wider than the interval around point estimate 0.10.
7.	Descriptive Statistics in Excel was used to calculate the mean and standard deviation of a sample of hours worked per week by 15-year-old high school students. The mean hours worked was 7.15 and the standard deviation was 1.67. If you wish to construct a 97% confidence interval for the mean number of hours worked by all 15-year-olds that is no more than half an hour wide, how many students would you have to survey? Describe your method of finding the answer to this question with a step by step list of choices, Excel commands and their associated intermediate values.
Answer	<p>The margin of error formula will be used to solve for minimum sample size required. Since the standard deviation at hand is a sample standard deviation, the student's t distribution would be used to calculate the interval. However, the student's t distribution depends on sample size to determine its value (n-1), hence must use the Z distribution and emphasize that the resulting number is the MINIMUM sample size.</p> <ol style="list-style-type: none"> 1. Calculate the margin of error: since the desired interval is supposed to be one-half hour wide, the margin of error is one-quarter of an hour, or 0.25. 2. The standard score used in this calculation will be a Z-score (see above for explanation) and it will be equal to $=\text{NORMSINV}(0.985) = 2.17009038$ $e = Z_{\frac{\alpha}{2}} * \frac{s}{\sqrt{n}} = 0.25 = 2.17009038 * \frac{1.67}{\sqrt{n}} \Rightarrow$ <ol style="list-style-type: none"> 3. $n = \left(\frac{Z_{\frac{\alpha}{2}} * s}{e} \right)^2 = \left(\frac{2.17009038 * 1.67}{0.25} \right)^2 = 210.1399 \approx 211$
8.	Suppose you calculated a 95% confidence interval for the true proportion of brown M&M's in the standard mix sold to the public. The interval was calculated to be [0.08,0.42]. Interpret this interval completely and accurately.
Answer	I am 95% confident that the true proportion of brown M&Ms in the standard M&Ms mix sold to the public is between 8% and 42%.