

Suggested Answers for WarmUps for Lesson 18

1.	How would you tell if you had a population standard deviation or a sample standard deviation? List as many clues as you can think of.
Answer	<p>Some things to look for are:</p> <ol style="list-style-type: none"> 1. if the data are reported to be from a sample. 2. if the data are reported to be from a population, or you have reason to believe you have all the available data. 3. if it says that the standard deviation is from a population (or a sample.) 4. if σ is used or if s is used to represent it.
2.	How would you tell if you needed to use a standard error? List as many clues as you can think of.
Answer	<p>Some things to look for are:</p> <ol style="list-style-type: none"> 1. if the question is asking about a sample mean or a sample proportion. 2. if a sample size is given and an “average” value is given. 3. if a sample size is given and a number of successes is given AND the question specifies a sample proportion.
3.	<p>The probability of Z being between -1 and 2 is (select one)</p> <ol style="list-style-type: none"> 1. greater than 2. less than 3. equal to 4. greater than or equal to 5. less than or equal to 6. unpredictable when compared to <p>the probability of a Student's t random variable being between -2 and 1 for any finite n. Why?</p>
Answer	<p>...greater than . . . Because of the symmetry of both Z and t, the two random variables are considering probabilities associated with identical intervals. Thus, the question becomes how does the distribution of the probability under the curves change with the variables. For the Z, more probability lies in the center of the distribution than in the Student's t, which has more probability in the tails than does the Z. Hence, over any interval that covers the center of the distribution, the Z will have more probability than will the Student's t.</p>
4.	Twenty percent of wells drilled in areas deemed favorable strike oil. If S_i is the

	number of wells that strike oil out of 30 sites, let Q_j be defined: $Q_j = \frac{S_i}{30}$. What is the expected value of $Q(i)$? Explain how you figured it out.
Answer	<p>Given the information, S_i looks like a Binomial, where S_i is the number of successes. Thus, when S_i is divided by a number of trials, Q_j, the sample proportion, is created. The expected value of the sample proportion is the population proportion, as proven in class:</p> <ol style="list-style-type: none"> $E(p) = E(X/n) = (1/n) * E(X)$ Distributed E and rearranged function. $E(X) = n * \pi$ Evaluated the term. $E(p) = (1/n) * n * \pi = \pi$ <p>Thus, the expected value of Q_j is 0.2 (20%)</p>
5.	Twenty percent of wells drilled in areas deemed favorable strike oil. If S_i is the number of wells that strike oil out of 30 sites, let Q_j be defined: $Q_j = \frac{S_i}{30}$. What is the variance of $Q(i)$? Explain how you figured it out.
Answer	<p>Again, given the information, S_i looks like a Binomial, where S_i is the number of successes. Thus, when S_i is divided by a number of trials, Q_j, the sample proportion, is created. The Variance of a sample proportion is $[\pi * (1 - \pi)] / n$, as proven in class.</p> <ol style="list-style-type: none"> $V(p) = V(X/n) = (1/n)^2 * V(X)$ Distributed V and rearranged the function, squaring the constant n. $V(X) = n * \pi * (1 - \pi)$ Evaluated the variance of X. $V(p) = (1/n)^2 * n * \pi * (1 - \pi) = (n/n^2) * \pi * (1 - \pi) = [\pi * (1 - \pi)] / n$ Substituted the values into the function and solved. <p>Thus, the variance of Q_j is $(0.2 * (1 - 0.2)) / 30$ or 0.00533.</p>
6.	According to research carried out by the Institute for Sodor Railroad Development, freight trains on Sodor Island carry 120 cargo cars. In actual fact, the number of cargo cars carried by a train is a normally distributed random variable with a mean of 110 and a standard deviation of 40. Which will be greater, the probability that the mean number of cargo cars will be at most 115 for samples of $n=100$ or the probability that the mean number of cargo cars will be at most 120 for samples of $n=25$? Write down your Excel command as well as your answer.
Answer	<p>$=\text{NORMDIST}(115, 110, 4, 1) = 0.894350226$ (4 is standard error, or $40/\text{SQRT}(100)$) $=\text{NORMDIST}(120, 110, 8, 1) = 0.894350226$ (8 is standard error, or $40/\text{SQRT}(25)$)</p>
7.	If you saw the previous question on an exam, when you did not have access to Excel, how would you answer it? (Yes, it is possible, and, yes, it does show up on exams.) What could you calculate that would enable you to answer the question?
Answer	<p>Standardize both the 115 and the 120 and note that they are identical Z scores. $Z_{115} = (115 - 110) / (40/\text{SQRT}(100)) = 1.25$</p>

	$Z_{120} = (120-110) / (40/\text{SQRT}(25)) = 1.25$
8.	If X is a normally distributed random variable with a mean of 20 and a standard deviation of 3, what is the probability that the mean of a sample of size n=36 will be at least 18? Explain how you would answer this question WITHOUT USING EXCEL.
Answer	<p>This is answered the same way as the previous question. Standardize 18.</p> $Z_{18} = \frac{(18 - 20)}{3/\sqrt{36}} = -4$ <p>Since we knew the variable was normal to begin with, the Normal Rule applied, and we would expect 99.7% of the probability above -3 standard deviations. We looking for the area to the right of -4 standard deviations, which must be even closer to 1 or 100%.</p>