

Suggested Answers for WarmUps for Lesson 15

1.	Explain carefully and completely the relationship between the Standard Normal and the Student's t distributions. Be sure to discuss all elements of the distributions.
Answer	<p>The Student's t is an approximation of the Standard Normal utilized whenever population standard deviation is not known. Both distributions are symmetric at zero and "bell-shaped", although the t is often described as "mound-shaped". This refers to the fact that, due to additional uncertainty caused by not knowing the population standard deviation, the probability under the curve is distributed slightly differently, in that more is found in the tails and less in the center of the distribution. Thus, in order to capture, say, 60% of probability symmetric about the center of the distribution, the interval over which this 60% will be found is wider for the t than for the Standard Normal. Both distributions are measured in # of standard deviations away from the mean, although the standard deviation of the Student's t distribution is always larger than 1. (df/(df-2))</p>
2.	According to the Center for Client Retention, 40% of all callers to automated customer-service systems automatically opt to go to a live operator when given the chance. The company you represent averages 1796 calls per day. Explain in detail how you would use the Normal to approximate the Binomial when you wish to calculate the probability that more than 1200 calls per day opt to go to a live operator.
Answer	<ol style="list-style-type: none"> 1) First calculate expected values for the Binomial described, with $n=1796$ and $\pi = 0.4$. 2) Determine exactly which values of X are included in the area of interest, by sketching a number line, indicating values of X below and above 1200 and determining if they belong in the area. 3) Decide how the continuity correction factor should be applied. This depends on the result of 2 above, and will be either the adding or subtracting of 0.5 to/from 1200. 4) Write the Excel command and calculate the probability.
3.	According to the Center for Client Retention, 40% of all callers to automated customer-service systems automatically opt to go to a live operator when given the chance. The company you represent averages 1796 calls per day. Use the Normal to approximate the Binomial to calculate the probability that more than 1200 calls per day opt to go to a live operator. Respond with your Excel command as well as your numerical answer.
Answer	<ol style="list-style-type: none"> 1) $E(X) = 1796 \cdot 0.4 = 718.4$ $SD(X) = \text{SQRT}(1796 \cdot 0.4 \cdot 0.6) = 20.7615$. 2) The phrase "... the probability that more than 1200 calls ..." suggests that the X values along the number line that ARE included in the area of interest are those that are GREATER than 1200 and specifically do NOT include 1200. 3) The continuity correction factor would be applied by adding 0.5 to 1200, to get 1200.5 as the place for the area to begin when calculated by the Normal. By being slightly larger than 1200, none of the area associated with 1200 is included in the

	<p>calculation.</p> <p>4) =1-NORMDIST(1200.5,718.4,20.7615,1) = 0 (practically impossible!)</p> <p>5) How about more than 750? =1-NORMDIST(750.5,718.4,20.7615,1) = 0.061036459</p>
4.	Explain the circumstances under which the Student's t distribution is used. What must be known or assumed in order to use the Student's t?
Answer	The Student's t distribution is used whenever the population standard deviation is not known for a continuous distribution. The population from which the sample was drawn must be nearly normal or be assumed to be normal.
<p>The time, T, it takes to give a man a shampoo and a haircut is a normally distributed random variable with a mean of 22 minutes and a standard deviation of 3 minutes. Suppose three men were selected randomly from among the male customers at a certain shop. Define $\bar{Q}_a = \frac{1}{3}T_i + \frac{1}{3}T_j + \frac{1}{3}T_k$ where each time, T, is from the same distribution described above. Use this for questions 5 and 6.</p>	
5.	Explain the steps you take to calculate the expected value of Q-bar. What is the expected value of Q-bar?
Answer	<p>1) Distribute the expectations operator over the function: $E(Q\text{-bar}_a) = E(\frac{1}{3}T_i + \frac{1}{3}T_j + \frac{1}{3}T_k)$</p> <p>2) Pull out the common value of $\frac{1}{3}$: $E(Q\text{-bar}_a) = \frac{1}{3}E(T_i + T_j + T_k)$</p> <p>3) Distribute the expectations operator some more: $E(Q\text{-bar}_a) = \frac{1}{3}(E(T_i) + E(T_j) + E(T_k))$.</p> <p>4) Substitute the expected value of T: $E(Q\text{-bar}_a) = \frac{1}{3}(22+22+22)$</p> <p>5) Do the math: $E(Q\text{-bar}_a) = \frac{1}{3}(66) = 22$</p>
6.	Describe the steps you would follow to calculate the standard deviation of Q-bar. What is the standard deviation of Q-bar?
Answer	<p>Remember that to get standard deviation, the variance must be calculated.</p> <p>1) Distribute the variance operator over the function: $V(Q\text{-bar}_a) = V(\frac{1}{3}T_i + \frac{1}{3}T_j + \frac{1}{3}T_k)$</p> <p>2) Distribute the variance operator some more: $V(Q\text{-bar}_a) = (V(\frac{1}{3}T_i) + V(\frac{1}{3}T_j) + V(\frac{1}{3}T_k))$.</p> <p>3) Distribute the variance operator some more: $V(Q\text{-bar}_a) = ((\frac{1}{3})^2V(T_i) + (\frac{1}{3})^2V(T_j) + (\frac{1}{3})^2V(T_k))$.</p> <p>4) Pull out the common value of $(\frac{1}{3})^2$: $V(Q\text{-bar}_a) = (\frac{1}{3})^2(V(T_i) + V(T_j) + V(T_k))$</p> <p>5) Substitute the variance of T = $3^2 = 9$: $V(Q\text{-bar}_a) = (\frac{1}{3})^2(9+9+9)$</p> <p>6) Do the math: $V(Q\text{-bar}_a) = (1/9)(9+9+9) = 1/9*27=27/9 = 3$</p> <p>7) Take the square root of the variance: $=\text{SQRT}(3) = \frac{3}{\sqrt{3}}$</p>
7.	What is a sampling distribution? Explain carefully, please.

Answer	A sampling distribution is the distribution of a statistic. Specifically, like all random variables, statistics calculated from random samples have distributions. The sampling distribution is the center, spread and shape created by all possible values of the statistic calculated from samples of the same size drawn randomly from the same population.
8.	Study the graph on the top of page 177 in your course work book. Compare and contrast the distribution of the two variables in that graph. Be complete.
Answer	Individual pulse rates present a very flat graph, almost uniform in shape (discounting the two “peaks” at 55-60 and 70-75,) while the graph of mean pulse rates is very triangular and could almost be described as “normal”. The “balance point” of the two distributions appears to occur in the same class, 70-75. Based on the range, the spread of pulse rates is larger than the spread of mean pulse rates, indicating that there is more variation among individuals than among means.