

Suggested Answers for WarmUps for Lesson 14

The time, T , it takes to give a man a shampoo and a haircut is a normally distributed random variable with a mean of 22 minutes and a standard deviation of 3 minutes. Suppose three men were selected randomly from among the male customers at a certain shop.

Define $Q_i = T_i + T_j + T_k$ where each time, T , is from the same distribution described above.

1.	What is the expected value of "Q"? Write down the steps you would take to calculate it.
Answer	$E(Q) = E(T_i + T_j + T_k) = E(T_i) + E(T_j) + E(T_k) = 22 + 22 + 22 = 66$
2.	Calculate the variance of "Q". Write down the steps you followed to calculate it.
Answer	$V(Q) = V(T_i + T_j + T_k) = V(T_i) + V(T_j) + V(T_k) = 3^2 + 3^2 + 3^2 = 9 + 9 + 9 = 27$
According to research carried out by the Institute for Sodor Railroad Development, freight trains on Sodor Island carry 120 cargo cars. In actual fact, the number of cargo cars carried by a train is a normally distributed random variable with a mean of 110 and a standard deviation of 40. Use this information to answer the following three questions.	
3.	What is the maximum number of cargo cars carried by the middle 70% of freight trains? Write down the Excel function as well as the answer to the question.
Answer	$=\text{NORMINV}(0.85, 110, 40) = 151.4573$ or 152
4.	What is the Z-score of the number of cargo cars carried by a Sodor Island train as reported by the Institute for Sodor Railroad Development? Describe your calculation.
Answer	The Institute for Sodor Railroad Development reports that trains carry 120 cargo cars. The Z score for this value is 120 minus the mean of the distribution, 110. The difference, 10, is divided by the standard deviation of 40 to give a Z-score of 0.25.
5.	What is the minimum number of cargo cars carried by the longest 23% of Sodor Island trains? Write down your Excel function as well as your answer.
Answer	$=\text{NORMINV}(1 - 0.23, 110, 40) = 139.5539$ or 140.
Nationwide, the probability of a twin birth is 0.02. For the following questions consider a set of 2,000 live births (a live birth means that a birthing process resulted in at least one live infant.) Use the normal to approximate the binomial probabilities requested in the next two questions.	
6.	What is the likelihood that of the 2,000 live births, there were at least 50 sets of twins? Write down the Excel function you used as well as your answer.
Answer	$E(X) = 0.02 * 2000 = 40$ $V(X) = 0.02 * 0.98 * 2000 = 39.2$ $\sigma = 6.261$ $= 1 - \text{NORMDIST}(49.5, 40, 6.261, 1) = 0.064592$

7.	What is the probability that there were fewer than 35 sets of twins in 2,000 live births? Write down your Excel function as well as your answer.
Answer	$E(X)=0.02*2000 = 40$ $V(X) = 0.02*0.98*2000 = 39.2$ $\sigma = 6.261$ $=NORMDIST(34.5,40,6.261,1) = 0.189849$
8.	Using your knowledge of linear combinations, if the sample proportion, p , is defined as X/n , when X is a binomial random variable, what is the expected value and the expected variance of p ? Write your answer as a "documented solution" which lists each step taken in the solution and the intermediate answers.
Answer	<ol style="list-style-type: none"> 1. $E(p)= E(X/n) = (1/n)*E(X)$ Distributed E and rearranged function. 2. $E(X) = n*\pi$ Evaluated the term. 3. $E(p)=(1/n)*n*\pi = \pi$ Substituted and cancelled n. <ol style="list-style-type: none"> 1. $V(p)=V(X/n) = (1/n)^2*V(X)$ Distributed V and rearranged the function, squaring the constant n. 2. $V(X)=n*\pi*(1-\pi)$ Evaluated the variance of X. 3. $V(p)=(1/n)^2*n*\pi*(1-\pi) = (n/n^2)*\pi*(1-\pi) = [\pi*(1-\pi)]/n$ Substituted the values into the function and solved.