

**E 370 – Fall 2002
In-Lab Examination
Exam Four--White**

Statement of Academic Integrity:

“I swear that I have neither given nor received assistance on this exam and that I will not discuss this exam until all sections have completed it, that is until 14:00 on Saturday, November 23, 2002.”

I have read and agree with the above statement.

Signature: _____

Name: (please print) _____

Team Number _____

Instructions

1. Write your name on every page of your exam.
2. Answer the questions in the spaces provided on the exam.
3. Do not provide computer output unless it is required. Whenever you print out anything, you **MUST TYPE** your Name and 5-digit Team Number on your work before you print it.
4. You must show your work and explanation to receive full credit. You must write any EXCEL functions used, with arguments, as well as the numerical output. For example, =NORMDIST(7,3,2,1) = 0.9773. Include 4 decimal places in your answer.
5. You may only use EXCEL for this exam.
6. Only the exam, pencils, erasers, and the tool cards may be on your desk. Put all the rest of your belongings along the wall or at the front of the room.
7. Remember, a student is to avoid even the appearance of cheating. Keep your eyes on your exam or on your computer screen. Any questionable behavior on your part is sufficient reason for me to confiscate your exam and ask you to leave the room.
8. The value of each question is given by each question. Budget your time accordingly.
9. When you are finished, you may turn in all pages of the exam and leave the room as soon as you are able to without disturbing your classmates.
10. Stay calm and do your best.

1. A student is taking a multiple-choice quiz in which each question has four choices. Suppose that she has no clue of the correct answers to any of the questions, and thus decides on a strategy in which she will place four balls (marked A, B, C, and D) into a box. She randomly selects one ball for each question and replaces the ball back in the box. The marking on the ball selected will determine her answer to the question.

(a) If there are five MC questions of four options on the exam, what is the probability that she will get:

(1) five questions correct? (2 points)

$$= \text{BINOM.DIST}(5,5,0.25,0) = 0.0010$$

(2) at least four questions correct? (2 points)

$$= 1 - \text{BINOM.DIST}(3,5,0.25,1) = 0.0156$$

(3) no questions correct? (2 points)

$$= \text{BINOM.DIST}(0,5,0.25,0) = \text{BINOM.DIST}(0,5,0.25,1) = 0.2373$$

(4) no more than two questions correct? (2 points)

$$= \text{BINOM.DIST}(2,5,0.25,1) = 0.8965$$

(b) What assumptions are necessary in (a)? (4 points)

That each outcome is independent of any other outcome, and that all balls are identical with the exception of the letter marked on the ball.

(c) What are the average and the standard deviation of the number of questions that she will get correct? (4 points)

$$E(X) = 5 \cdot 0.25 = 1.25$$

$$V(X) = 5 \cdot 0.25 \cdot (1 - 0.25) = 0.9375 \rightarrow \text{St. Dev} = 0.9682$$

(d) Suppose the exam has 50 multiple-choice questions and 30 or more correct answers is a passing score. What is the probability that she will pass the exam? (4 points)

$$= 1 - \text{BINOM.DIST}(29,50,0.25,1) = 1.64229E-07 = 0$$

2. A study of 104 corporations found that the salaries of their CEOs had increased an AVERAGE of 6.9% last year, with a standard deviation of 17.4%.
- (a) The 104 individual percent increases form a right-skewed distribution. Explain why we can nonetheless act as if the mean increase has a normal distribution. (3 points)

Because the number of items in the sample is quite large, specifically larger than 29, the Central Limit Theorem assures us that the means of such samples will be approximately normally distributed.

- (b) Set up a 95% confidence interval for the national average percent increase in the salaries of CEOs. (8 points)

$$6.9\% \pm T.INV(0.975,103) * \left(\frac{17.4}{\sqrt{104}} \right) = 6.9\% \pm 3.3839$$

- (c) Provide an interpretation of the confidence interval constructed in (b). (3 points)
I am 95% confident that the population mean increase in CEO salaries last years is 6.9% plus or minus 3.3839%.

- (d) If a student wrote down $6.9\% \pm 1.55 \times SE(\bar{X})$ to compute the confidence interval, which α value was she using? (3 points)

$$=2*(1-T.DIST(1.55,103,1))= 0.1242$$

- (e) If you want to get a narrower confidence interval than you had in (b), what strategy would you take? (3 points) **Either reduce the level of confidence or increase the size of the sample.**

3. IU was reported as the nation's No.1 party school. As a result, some people think that the average credit hours taken by IU students per semester must be lower than the national average of 12 credit hours. The following Excel printout contains some descriptive statistics for credit hours from a sample of 36 IU students. At the 5% level of significance, do you think IU students' average credit hours are less than national average credit hours?

Mean	11.8
Sample Variance	5.7333
Count	36

- (a) State the null hypothesis and the alternative hypothesis. (4 points)

$$H_0: \mu \geq 12$$

$$H_1: \mu < 12$$

- (b) Calculate the critical value. (4 points)

$$=T.INV(0.05,35) = 1.$$

- (c) State the decision rule using the results in part (b). (4 points)

If $t_{obs} < -1.6899$, reject the null hypothesis; otherwise fail to reject the null.

- (d) Calculate the test statistic. (4 points)

$$t_{obs} = \frac{11.8 - 12}{\sqrt{\frac{5.7333}{36}}} = -0.5012$$

- (e) Make a decision based on the decision rule in (c) and interpret your result or making conclusion statement. (4 points)

Since -0.5012 is NOT less than -1.6899, I fail to reject the null. There is no evidence that the mean credit hours of IU students is different from the nation average.

4. Green Transit claims that 40% of its buses are on time. A sample of 100 bus arrivals observed last week revealed that 43 arrived on time. At the 0.10 significance level, can we conclude that GT understates its performance?

(a) State the null and alternative hypotheses. (4 points)

$$H_0: \pi \leq 0.40$$

$$H_1: \pi > 0.40$$

(b) Write a decision rule using p-values. (4 points)

If the p-value from the observed sample value is less than alpha = 0.10, reject the null; otherwise fail to reject the null.

(c) Calculate the test statistic. (4 points)

$$Z_{obs} = \frac{0.43 - 0.40}{\sqrt{\frac{0.40 * (1 - 0.40)}{100}}} = 0.6124$$

(d) Calculate the p-value for this test and state your conclusion. (4 points)

$$\mathbf{p\text{-value} = 1 - \text{NORM.S.DIST}(0.6124, 1) = 0.2701}$$

(e) According to your results, if you want to reject the null hypothesis, what would be the lowest value of α you could choose? (4 points—extra points)

Alpha must be just larger than 0.2701 in order to reject the null hypothesis based on this evidence.

5. According to Fresh Milk's specification, the amount of milk in its 1-gallon FreshCal Milk bottles has a standard deviation of 0.02 gallon. A random sample of 1-gallon FreshCal Milk was selected. The following table contains some Excel-generated descriptive statistics for this sample. The values are in gallons.

Mean	0.995
Minimum	0.930
Sum	99.5

- (a) Set up a 95% confidence interval for the true average amount of milk contained in a 1-gallon bottle, based on the information given above. Show how you obtained it using the information and the relevant Excel functions. (8 points)

Sum/count = Mean, thus $99.5/0.995 = 100$, thus $n=100$

$$0.995 \pm \text{NORM.S.INV}(0.975) * \frac{0.02}{\sqrt{100}} = 0.995 \pm 1.96 * 0.002 = 0.995 \pm 0.0039$$

The interval is (0.9911, 0.9989)

- (b) We want to test the null hypothesis that the true mean of amount of milk is 1 gallon. What will be the result of the test based on the result of (a) and why? (4 points)

Reject the null. The interval calculated at the 95% level, ie at $\alpha = 0.05$ does not include the value of one gallon.

- (c) If the level of confidence was 99%, will you change your decision in (b) using your knowledge of confidence interval? (8 points)

Possibly, because increasing the level of confidence will widen the interval. The 1.96 in the calculation in (a) becomes $=\text{NORM.S.INV}(0.995) = 2.5758$. Recalculation is necessary to be sure.

$$2.5758 * 0.002 = 0.0052$$

$0.995 + 0.0052 = 1.0002$. At the 1% level we would fail to reject the null.