1. The life times of light bulbs that are advertised to last for 5000 hours are normally distributed with a mean of 5100 hours and a standard deviation of 200 hours.

(a) What is the probability that a randomly selected bulb lasts longer than the advertised figure and shorter than 5250 hours? (3 points)

Ans: P(5000<X<5250)=NORM.DIST(5250,5100,200,1)- NORM.DIST(5000,5100,200,1)

= 0.4648

(b) A selected bulb belongs to the best 10% of all bulbs. What is the minimum possible life time for such a bulb? (2 points)

Ans: NORM.INV(1-.1, 5100, 200)= 5356.3103(hours)

Suppose we do not know the population standard deviation. The population mean is still 5100 hours. A sample of 100 bulbs is selected. The sample standard deviation is 210 hours. Finish (c), (d) and (e) under this assumption.

(c) What is the probability that a randomly selected bulb lasts longer than the advertised figure? (6 points)

Ans: t-score of 5000=(5000-5100)/210= -0.476190479

P(X>5000)=1-T.DIST(-0.476190479,99,1)= 0.6825

(d) What is the maximum possible life time of a bulb which belongs to the middle 70% of all bulbs? (3 points)

Ans: =T.INV(0.85,99)=1.041891

Min=5100+1.041891*210=5318.797 (or 5318.4)

(e) Let X to be a random variable which represents the life of a bulb. When P(4900<X<c)=0.7, what is the value of c? (4 points)

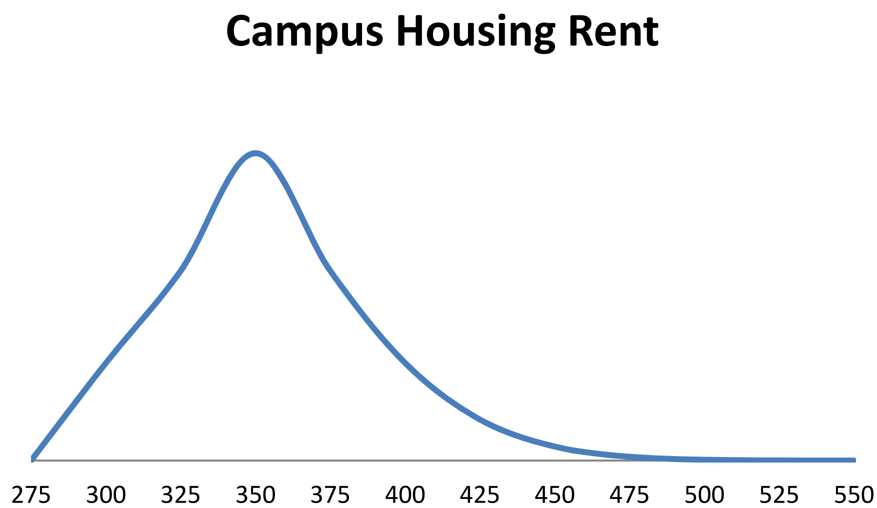
T score of 4900 = (4900-5100)/210 = -0.95238

Prob(x<4900) = T.DIST(-0.95238,99,1)= 0.171612,

t score of c=T.INV((0.171612+0.7),99)= 1.140629,

c=1.140629*210+5100=5339.532

2. Sixty-five percent of IU students live in on-campus apartments or dorms, and the rent for each individual has a mean of $350 and a standard deviation of $50.



(a) A sample of 25 students who live on campus is selected. What is the probability that the sample mean rent equals $350? What is the probability that the sample mean rent is between $310 and $360? Interpret your answers.(7 points)

Ans: Part I: P(x-bar=350)=0

Part II: We do not know P(310<x-bar<360), because the sample size is less than 30, it's difficult to tell the distribution the sample mean follows.

(b) How would your results differ from part (a) if you worked with samples of size 50 instead? Be very specific. (6 points)

Ans: Part I: P(X-bar=350)=0

PartII.P(310<x-bar<360)= NORM.DIST(360,350, 50/SQRT(50),1)-NORMDIST(310,350,50/SQRT(50),1) =0.9213

We can use normal distribution to estimate the sample mean by central limit theory, because the sample size is greater than 30.

(c) The university housing office wants to estimate how many students live on campus. Suppose we select a sample of 40 IU students. When P(c<p<0.75)=0.2, what is the value of c? (6 points)

Ans: Standard error of p= SQRT(0.65*(1-0.65)/40) =0.075415516

P(p<0.75)=NORM.DIST(0.75,0.65, 0.075415516,1)= 0.907578013;

c= NORM.INV((0.907578013-0.2), 0.65, 0.075415516) = 0.6912

3. Suppose you are the marketing manager of a detergent company. Your company developed a new kind of detergent with a jasmine scent. You would like to see how this new detergent is

accepted by local residents. You randomly asked consumers in front of a WalMart store, and only 8 out of 20 consumers said that they liked this product.

(a) What is your point estimate? (2 points)

The point estimate is sample proportion, p = 8/20=0.4.

(b) Construct a 90% confidence interval and report your upper and lower limits. (7 points)

0.4 ± NORM.S.INV(0.95))*SQRT((0.4*0.6)/20)] = [0.2198, 0.5802]

(c) Interpret the interval you have constructed (2 points)

With 90% confidence, I believe that the true proportion (or population proportion) of consumers who would like this product is between 21.98% and 58.02%.

(d) If you would like to make the width of the confidence interval less than 30%, what confidence level should you choose? (7 points)

$$e \le \frac{0.3}{2} = 0.15$$

$$e = Z_{\frac{\alpha}{2}} \times \sqrt{\frac{0.4 \times 0.6}{20}} = Z_{\frac{\alpha}{2}} \times 0.109545 = 0.15 \rightarrow Z_{\frac{\alpha}{2}} = \frac{0.15}{0.109545} = 1.369306$$

⇨ $\frac{\alpha}{2} = prob(Z > 1.369306) \rightarrow \alpha = 2\times\ prob(Z > 1.369306)$

⇨ α **= 2× [1- NORM.S.DIST(1.369306)] = 0.170904**

⇨

⇨ **confidence level = 1-** α **= 0.829096≈0.8291**

➔ **So, any confidence level below 82.91% will result in a confidence interval narrower than 30%.**

4. The following table is partial output from Excel's Descriptive Statistics. The data are the fill of 2-liter bottles of soft drink. Operations control is concerned that the profit margin is eroding and wants to know if their two-liter bottles are overfilled. Use this information to answer the next questions.

Amount	
Standard Deviation	**0.0565**
Sample Variance	**0.00319**
Sum	200.93
Count	100

(a) Perform a hypothesis test using all steps to determine if the fill process is overfilling the two-liter bottle. Suppose $\alpha = 0.05$ and use the p-value method for the test.

Step 1. Write all hypotheses (4 points)

H_o : μ ≤ 2

H_1 : μ > 2

Step 2. Write a decision rule using p-value method. (2 points)

If p_value < α ➔ reject Ho ; Otherwise, fail to reject Ho

Step 3. Calculate the test statistic (2 points)

For α =0.05
t_{obs} = (2.0093-2)/(0.0565/SQRT(100)) =1.6460

Step 4. Calculate the p-value for this test and state your conclusion (5 points)

P-value = 1-T.DIST(1.6460, 99, 1) = 0.0515 > α
We fail to reject H_o because P-value >α. So there is not enough evidence that their two-liter bottles are overfilled.

(b) What would be the result of the test if it was known that **σ = 0.0565**? Why? (7 points)

(i) Z_{obs} = (2.0093-2)/(0.0565/sqrt(100)) = 1.6460
P-value = 1- NORM.S.DIST(1.6460) = 0.0499 < α
We can reject H_o because P-value <α . So there is enough evidence that their two-liter bottles are overfilled.

(ii) Since the Student's t distribution has fatter tails than the standard normal distribution, the p-value in part (b) is smaller than that in part (a).