Practice Hypothesis Tests

1. ATMs must have sufficient cash to satisfy customers' weekend withdrawals. One branch anticipates a withdrawal to be \$160 on average with σ = \$30. A sample of 36 withdrawals has a mean of \$172. At the 2% level, should this branch change the amount of cash in its ATM?

    a. H_0: $\mu \le$ \$160 H_1: μ>\$160
    b. α = 0.02
    c. Population information is given, so the test statistic will be the Standard Normal, Z. This is a right tailed test with one critical value =NORMSINV(0.98) = 2.054
    d. Decision Rules:
        i. Critical Value: If Z_{OBS} > ZCR = 2.054, reject the null and conclude that more cash needs to be added to the ATMs, otherwise fail to reject the null and make no changes.
        ii. p-value: If the p-value is less than alpha, reject the null and conclude that more cash needs to be added to the ATMs, otherwise fail to reject the null and make no changes.
    e. Test Statistic: $$Z_{OBS} = \frac{172 - 160}{30/\sqrt{36}} = 2.4$$
    f. Result of test: Z_{OBS} = 2.4 > 2.054, thus reject the null and conclude that the bank needs to increase the amount of cash in the ATMs for weekends.
    g. P-value =1-NORMSDIST(2.4) = 0.008197, which is, in turn smaller than alpha = 0.02.

2. In a sample of 2101 Vietnam Veterans, it was discovered that 777 had been divorced at least once. The US Census reported in 1985 that of all American men aged 30 to 44, 27% had been divorced at least once. At the 3% level, do Vietnam vets have a higher divorce rate?

    a. H_0: $\pi \le 0.27$ H_1: π>0.27
    b. α = 0.03
    c. The problem deals with a sample proportion, so if n*π AND n*(1-π) are both at least 5, the test statistic will be the Standard Normal, Z.
        i. n*π = 2101*0.27 = 567.27 > 5
        ii. n*(1-π) = 2101*(1-0.27) = 1533.73 > 5
        iii. This is a right tailed test with one critical value =NORMSINV(0.97) = 1.881
    d. Decision Rules:
        i. Critical Value: If $Z_{OBS} > Z_{CR}$ = 1.881, reject the null and conclude that Vietnam vets have a higher rate of divorce than American men as a whole, otherwise fail to reject the null and make no changes.
        ii. p-value: If the p-value is less than alpha, reject the null and conclude that Vietnam vets have a higher rate of divorce than American men as a whole, otherwise fail to reject the null and make no changes.
    e. Test Statistic: $$Z_{OBS} = \frac{0.3698 - 0.27}{\sqrt{\frac{0.27*(1-0.27)}{2101}}} = 10.3063$$
    f. Result of test: Z_{OBS} = 10.3063 > 1.881, thus reject the null and conclude that Vietnam vets have a higher rate of divorce than American men as a whole,
    g. P-value =1-NORMSDIST(10.3063) = 0.0 , which is, in turn smaller than alpha = 0.03.

3. A manufacturer claims that the capacity of a certain battery type is 140 ampere-hours. An independent consumer protection agency wishes to test the credibility of the manufacturer's claim. A sample of 20 batteries had mean = 138.5, s = 2.7. What would the consumer protection agency decide at a level of 4%?

    a. H_0: μ = 140 hours H_1: μ … 140 hours
    b. α = 0.04
    c. The problem deals with a sample mean and population information is not known. Thus, the test statistic will be Student's t with 19 degrees of freedom. This is a two-tailed test with two critical values = ± TINV(0.04,19) = ± 2.205
    d. Decision Rules:

i. Critical Value: If | t_{OBS} | > | t_{CR} = 2.205| reject the null and conclude that the capacity of the battery is NOT 140 hours, otherwise fail to reject the null and go pick on another company.
ii. p-value: if the p-value is less than 0.04, reject the null and conclude that the capacity of the battery is NOT 140 hours, otherwise fail to reject the null and go pick on another company.

e. Test Statistic: $t_{OBS} = \dfrac{138.5 - 140}{2.7/\sqrt{20}} = -2.485$

f. Result of test: | -2.485| > | 2.205|, hence, reject he null and conclude that the capacity of the battery is NOT 140 hours.
g. p-value: =TDIST(2.485,19,2) = 0.022461, which is smaller than α = 0.04, thus, reject the null as above.

4. The MacBurger restaurant chain claims that the mean waiting time of customers for service is normally distributed, with a mean of 3 minutes and a standard deviation of 1 minute. The quality-assurance department found in a sample of 50 customers at the Warren Road MacBurger that the mean waiting time was 2.75 minutes. At the 0.06 significance level, can we conclude that the mean waiting time is less than 3 minutes?

a. H_0: $\mu \geq 3$ minutes H_1: $\mu < 3$ minutes
b. α = 0.06
c. This is a sample mean problem which deals with population information, hence the test statistic will be a Standard Normal, Z. This is a left tailed test with one critical value = NORMSINV(0.06) = -1.5548.
d. Decision Rules:
i. Critical Value: If $Z_{OBS} < Z_{CR}$ = -1.5548, reject the null and conclude that the mean waiting time is less than 3 minutes, otherwise fail to reject the null because there is insufficient evidence that the mean waiting time has decreased.
ii. p-value: If the p-value is < α = 0.06, reject the null and conclude that the mean waiting time is less than 3 minutes, otherwise fail to reject the null because there is insufficient evidence that the mean waiting time has decreased.

e. Test Statistic: $Z_{OBS} = \dfrac{2.75 - 3.0}{1/\sqrt{50}} = -1.768$

f. Result of test: Since -1.768 < -1.5548, reject the null and conclude that the mean waiting time is less than 3 minutes.
g. p-value: =NORMSDIST(-1.768) = 0.03855, which is smaller than 0.06, so, reject the null as above.

5. Indiana University tells its newly admitted students that the costs of textbooks are $420 on an average per semester. A sample of 100 students enrolled in the university shows a sample average of $430 with a standard deviation of $45. Using 7% level of significance, test the validity of the University's claim.

a. H_0: μ = 420 hours H_1: μ … 420 hours
b. α = 0.07
c. This problem deals with a sample mean without population information. Thus, the test statistic is a student's t with 99 degrees of freedom. This is a two-tailed test with two critical values = ± TINV(0.07,99) = ± 1.832.
d. Decision Rules:
i. Critical Value: If |t_{OBS}| > |t_{CR} = 1.832|, reject the null and conclude that the mean cost of textbooks per semester is not $420 hours, but has changed, otherwise fail to reject the null and don't bother updating the Orientation materials.
ii. p-value: If the p-value is less than 0.07, reject the null and conclude that the mean cost of textbooks per semester is not $420 hours, but has changed, otherwise fail to reject the null and don't bother updating the Orientation materials.

e. Test Statistic: $t_{OBS} = \frac{430 - 420}{45/\sqrt{100}} = 2.222$

f. Result of test: |2.222| > |1.832|, hence reject the null and conclude that the mean cost of textbooks per semester is not $420 hours, but has changed.

g. p-value: = TDIST(2.222,99,2) = 0.0285 < 0.07, thus, reject the null as above.

6. Developers will encounter environmentalist opposition if they attempt to expand into a forested area where the trees are more than 500 years old on average. They want to build a mall in Oregon on twenty acres of forest, and sample 30 trees to determine their ages. Of those 30 trees, the mean is 600 years with standard deviation 60 years. At 9% significance, can the developers build there without opposition?

a. H_0: $\mu \leq 500$ years H_1: $\mu > 500$ years

b. $\alpha = 0.09$

c. This is a problem using the sample mean but has no population information, hence, the test statistic is the student's t with 29 degrees of freedom. This is a right tailed test with one critical value = TINV(2*0.09,29) = 1.3739.

d. Decision Rules:

   i. Critical Value: If $t_{OBS} > t_{CR} = 1.3739$, reject the null and conclude that the trees are older than 500 years on average and realize that an attempt to build in this location will be met with environmental opposition, otherwise fail to reject the null and apply for permission to build in this location.

   ii. p-value: If the p-value is less than 0.09, reject the null and conclude that the trees are older than 500 years on average and realize that an attempt to build in this location will be met with environmental opposition, otherwise fail to reject the null and apply for permission to build in this location.

e. Test Statistic: $t_{OBS} = \frac{600 - 500}{60/\sqrt{30}} = 9.129$

f. Result of test: Since 9.129 is a whole lot bigger than 1.3739, reject the null and conclude that the trees are older than 500 years on average and realize that an attempt to build in this location will be met with environmental opposition.

g. p-value = TDIST(9.129,29,1) = 2.50296E-10 which is a whole lot smaller than 0.09.

7. The Roman Senate has become concerned about the loyalty of the army in Gaul commanded by Julius Caesar. They claim that the 50,000 men in the army, at least 20,000 are foreign barbarians. Caesar believes that there are fewer barbarians than that, so the Senate should not worry. He polls one legion, 1,000 men, and finds that 350 of them are barbarians. Is the Senatorial concern valid? Test at 8% significance.

a. H_0: $\pi \geq 20{,}000/50{,}000 = 0.40$ H_1: $\pi < 0.40$

b. $\alpha = 0.08$

c. This is a sample proportion question, which means that, if n*π AND n*(1-π) are both at least 5, the test statistic will be the Standard Normal

   i. n*π = 1000*0.4 = 400 > 5

   ii. n*(1-π) = 1000*(1-0.40) = 600 > 5

   iii. This is a left-tailed test with one critical value = NORMSINV(0.08) = -1.405.

d. Decision Rules:

   i. Critical Value: If $Z_{OBS} < Z_{CR} = -1.405$, reject the null and conclude that Caesar is correct and that there are fewer barbarians than the Senate feared, otherwise fail to reject and the Senate should continue to worry.

   ii. p-value: If the p-value is less than 0.08, reject the null and conclude that Caesar is correct and that there are fewer barbarians than the Senate feared, otherwise fail to reject and the Senate should continue to worry.

e. Test Statistic: $Z_{OBS} = \dfrac{0.35 - 0.40}{\sqrt{\dfrac{0.40 * (1 - 0.40)}{1000}}} = -3.227$

f. Test Result: Since -3.227 < -1.405, reject the null and conclude that Caesar is correct and that there are fewer barbarians than the Senate feared.

8. A new machine at a clothing factory is supposed to be producing cloth that has a mean breaking strength of 70 lbs with σ = 3.5 lbs. Management is concerned about lawsuits if the breaking strength is lower. A sample of 49 pieces had a mean of 69.1 lbs.

a. H_0: $\mu \geq 70$ lbs H_1: $\mu < 70$ lbs
b. α is not given, so can be selected to be whatever is desired. Select $\alpha = 0.01$.
c. This is a sample mean question with population information, hence the test statistic will be the Standard Normal. This is a left-tailed test with one critical value = NORMSINV(0.01) = -2.326.
d. Decision Rules:
   i. Critical Value: If $Z_{OBS} < Z_{CR} = -2.326$, reject the null and conclude that the machine is not operating properly and lawsuits are a definite possibility, otherwise, fail to reject the null and let the machine continue as currently set.
   ii. p-value: If the p-value is less than 0.01, reject the null and conclude that the machine is not operating properly and lawsuits are a definite possibility, otherwise, fail to reject the null and let the machine continue as currently set. **NOTE: This is an excellent opportunity to observe the value of the p-value method, since no α was pre-determined.**

e. Test Statistic: $Z_{OBS} = \dfrac{69.1 - 70}{3.5/\sqrt{49}} = -1.8$

f. Test Result: Since -1.8 is NOT less than -2.326, fail to reject the null and let the machine continue as currently set.
g. p-value: =NORMSDIST(-1.8) = 0.03593, which is NOT less than 0.01, so fail to reject the null as above. Note, for any $\alpha > 0.03593$, the decision would be reversed and the null would be rejected.

9. Dole Pineapple, Inc. is concerned that the 16-ounce can of sliced pineapple is being over filled. Historically, the standard deviation of the weight is 0.03 ounces. The quality-control department took a random sample of 50 cans and found that the arithmetic mean weight was 16.05 ounces. At the 5% level of significance, can we conclude that the mean weight is greater than 16 ounces?

a. H_0: $\mu \leq 16$ oz H_1: $\mu > 16$ oz
b. $\alpha = 0.05$
c. This is a sample mean question with population information, hence the test statistic is the Standard Normal. This is a right-tailed test with one critical value = NORMSINV(1-0.05) = 1.645.
d. Decision Rules:
   i. Critical Value: If If $Z_{OBS} > Z_{CR} = 1.645$, reject the null and conclude that the mean weight of the 16 oz can of pineapple is greater than 16 ounces, otherwise, fail to reject the null and allow the canning process to continue as currently set.
   ii. p-value: If the p-value is less than 0.05, reject the null and conclude that the mean weight of the 16 oz can of pineapple is greater than 16 ounces, otherwise, fail to reject the null and allow the canning process to continue as currently set.

e. Test Statistic: $Z_{OBS} = \dfrac{16.05 - 16}{0.03/\sqrt{50}} = 11.785$

f. Test Result: Since 11.785 is hugely greater than 1.645, reject the null and conclude that the mean weight of the 16 oz can of pineapple is greater than 16 ounces, and get the production process reset.
g. p-value: = 1-NORMSDIST(11.785) = 0. There is no alpha which can be smaller, so Dole should probably have looked at this problem much earlier. Again, reject the null as above.

10. General Electric manufactures jet engines and claims that they have a useful life of 2000 hours, standard deviation 100 hours. A sample of 50 engines had a mean useful life of 1800 hours. With 1% significance, is GE's claim valid?

a. H_0: μ = 2000 hours H_1: μ … 2000 hours
b. α = 0.01
c. This is a sample mean problem with population information, hence the test statistic is the Standard Normal. This is a two-tailed test with two critical values = ± NORMSINV(1-0.01/2) = ± 2.576.
d. Decision Rules
   i. Critical Value: If $|Z_{OBS}|$ > $|Z_{CR}$ = ± 2.576|, reject the null and conclude that the mean useful life of a GE jet engine is not 2000 hours, otherwise, fail to reject because there is insufficient evidence to indicate that the mean life is other than 2000 hours.
   ii. p-value: If the p-value is less than 0.01, reject the null and conclude that the mean useful life of a GE jet engine is not 2000 hours, otherwise, fail to reject because there is insufficient evidence to indicate that the mean life is other than 2000 hours.
e. Test Statistic: $$Z_{OBS} = \frac{1800 - 2000}{100 / \sqrt{50}} = -14.1421$$
f. Test Result: Since |-14.1421| > |2.576|, reject the null and conclude that the mean useful life of a GE jet engine is not 2000 hours.
g. p-value; = 2*(NORMSDIST(-14.142) = 2.08849E-45, an incredibly small number, confirming that the null is rejected, as above.