Preliminary

Measuring Capital Adjustment Costs by Dynamic Programming

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Prior Work

- Measurements of Quadratic Cost

  Shapiro, *QJE*, 1986 (GMM, Euler equation, manufacturing sector, quarterly data)

  Hall, *QJE*, 2004 (IV, Euler equation, industry level, annual data)

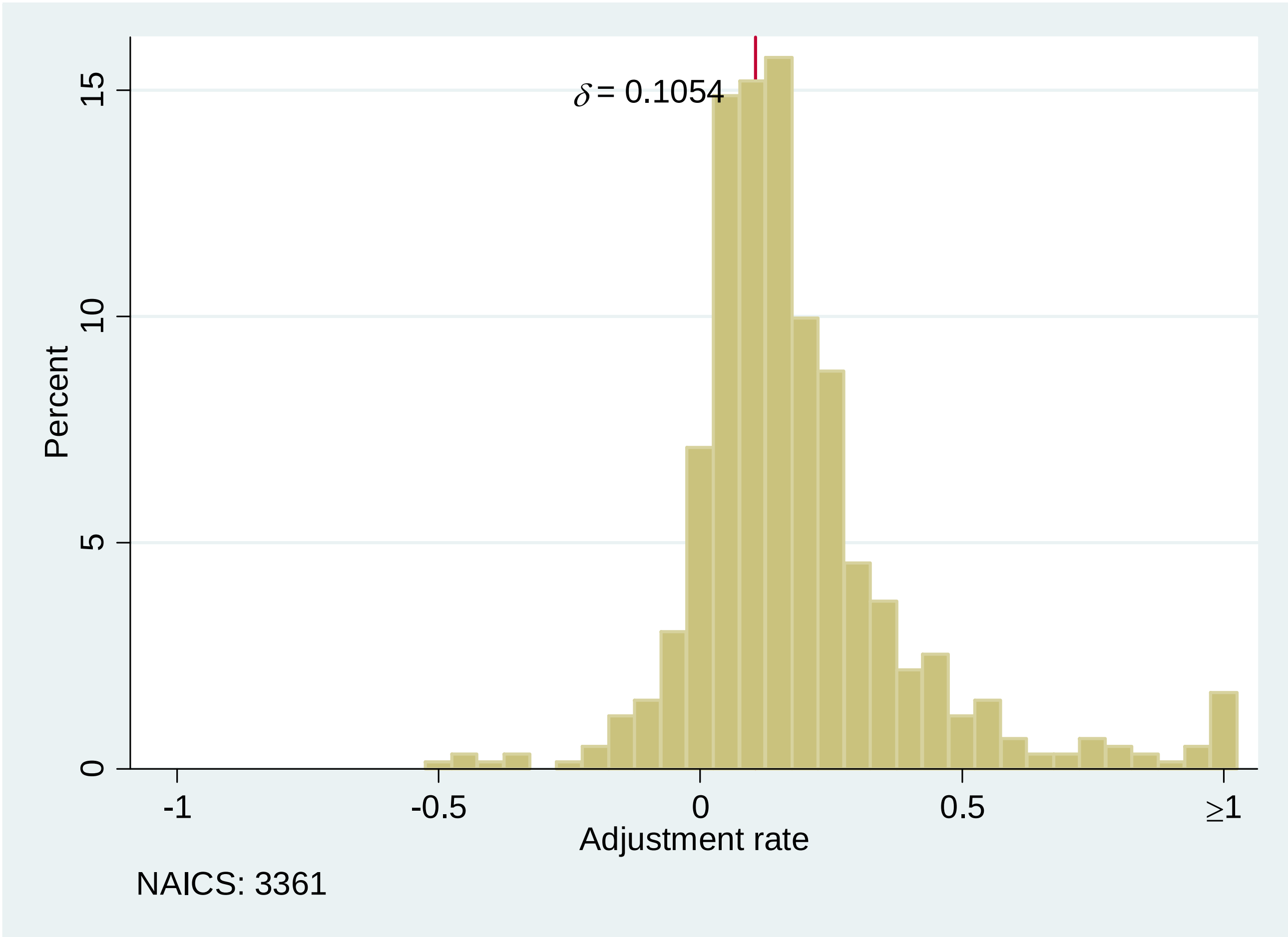
  Cooper and Haltiwanger, *RES*, 2006 (Indirect inference, Dynamic programming, plant level, annual data ?)

- Theoretical Model

  Abel and Eberly, *RES*, 1996

  Cooper and Haltiwanger, *RES*, 2006

Investment Rates



592 Observations, year 1980 to 2005

Theoretical Model (1)

- Linearly homogeneous Cobb-Douglas technology with 3 inputs (labor, intermediate goods, and capital stock) $(Q = Z_2 K^{1-\alpha-\gamma} L^{\alpha} M^{\gamma})$
- Frictionless adjustments in labor and intermediate goods
- (Linear and) Quadratic adjustment cost of capital adjustments
- One year of the time to build for capital
- Iso-elastic demand of output $(\varepsilon > 1)$
- Price taker for inputs

Theoretical Model (2)

- Firm's objective

$$\max_{\{K_t, L_t, M_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \pi(K_t, K_{t+1}, L_t, M_t | Z_A)$$

where

$$\pi(K_t, K_{t+1}, L_t, M_t | Z_A)$$
$$= P(Q_d | Z_1) \cdot Q(K_t, L_t, M_t | Z_2) - wL_t - qM_t$$
$$-\{K_{t+1} - (1-\delta)K_t\} - AC(K_t, K_{t+1})$$

K_0: given

$0 < \beta < 1$

Theoretical Model (3)

- FOC for L and M

  $w\tilde{L} = \alpha_{\varepsilon}(P \cdot Q)$ (Labor share)

  $q\widetilde{M} = \gamma_{\varepsilon}(P \cdot Q)$ (Intermediate goods share)

$$\pi(K_t, K_{t+1}, L_t, M_t | Z_A) =$$
$$Z_A K_t^{\zeta} - \{K_{t+1} - (1-\delta)K_t\} - AC(K_t, K_{t+1})$$

Bellman Equation

$$V[K, Z_A] = \max_{K'}\left[r(K, K', Z_A) + \beta E_Z V[K', Z_A']\right]$$

where

$$r(K, K', Z_A) =$$
$$-\{K' - (1-\delta)K\} - AC(K, K') + \beta E_Z[Z_A']K'^{\zeta}$$

(reward function)

Z_A is Markov chain $\Rightarrow E_Z[Z_A'] = \Pi^T Z_A$

Adjustment Costs Function (1)

- Functional forms

1) $1[I > 0]\{B_{up}I + C_{up}I^2\} + 1[I < 0]\{B_{dwn}|I| + C_{dwn}I^2\}$

2) $1[I > 0]\left\{B_{up}I + C_{up}\left(\frac{I}{K}\right)^2 K\right\} +$

$1[I < 0]\left\{B_{dwn}|I| + C_{dwn}\left(\frac{I}{K}\right)^2 K\right\}$

3) $1[I > 0]\left\{B_{up}\left(\frac{I}{K}\right) + C_{up}\left(\frac{I}{K}\right)^2\right\} +$

$1[I < 0]\left\{B_{dwn}\left(\frac{|I|}{K}\right) + C_{dwn}\left(\frac{I}{K}\right)^2\right\}$

Adjustment Costs Function (2)

- Gross investment

$$I = K' - (1 - \delta)K$$

or Net investment

$$I = K' - K$$

- 6 specifications of Adjustment Costs Function

Adjustment Costs Function (3)

Adjustment Costs Function		B_{up}	C_{up}	B_{dwn}	C_{dwn}	Other authors $C_{up} = C_{dwn}$
1	Gross	0	0.0009	0	0.0007	S: 0.00055 (0.00025)~0.0007 (0.0003)
	Net	0	0.0006	0	0.0003	
2	Gross	0	1.1	0	0.8	C&H: 0.0215 (0.0011), 0.0625 (0.0001) S: 0.105 (0.020)~0.125 (0.040)
	Net	0	1.0	0	0.8	Automotive industry H: 0.20 (0.65)
3	Gross	0	1,500	0	7,500	
	Net	0	1,500	0	1,500	

Standard errors in parentheses

Calibrations (1)

- Labor and intermediate goods share

	Revenue $P \cdot Q$	Annual Payroll wL	Cost of Materials qM	Labor share α_{ε}	Intermediate goods share γ_{ε}
1992	159,859,700	11,869,200	110,916,400	0.07425	0.69384
1997	220,052,857	13,022,700	147,707,890	0.05918	0.67124
2002	243,883,598	14,546,225	171,132,545	0.05964	0.70170
2007	262,236,645	13,619,094	188,632,977	0.05193	0.71932
			mean	<u>0.06125</u>	<u>0.69652</u>

Source: U.S. Census Bureau, "Economic Census" ($1,000)

Calibration (2)

- By assuming $\varepsilon = 7, \ \zeta = 0.1$

- Estimating Z_A; Z_A =(operating cash flow)/K^{ζ}

$$Z_A = (1.5736 \ \ 13.2403 \ \ 42.7360 \ \ 104.774 \ \ 274.909 \ \ 517.181 \ \ 1{,}209.66 \ \ 2{,}286.00 \ \ 4{,}165.05 \ \ 7{,}840.99)$$

Z_A is multiplied by a factor $D(\approx 2.5)$ in predictions for better fit.

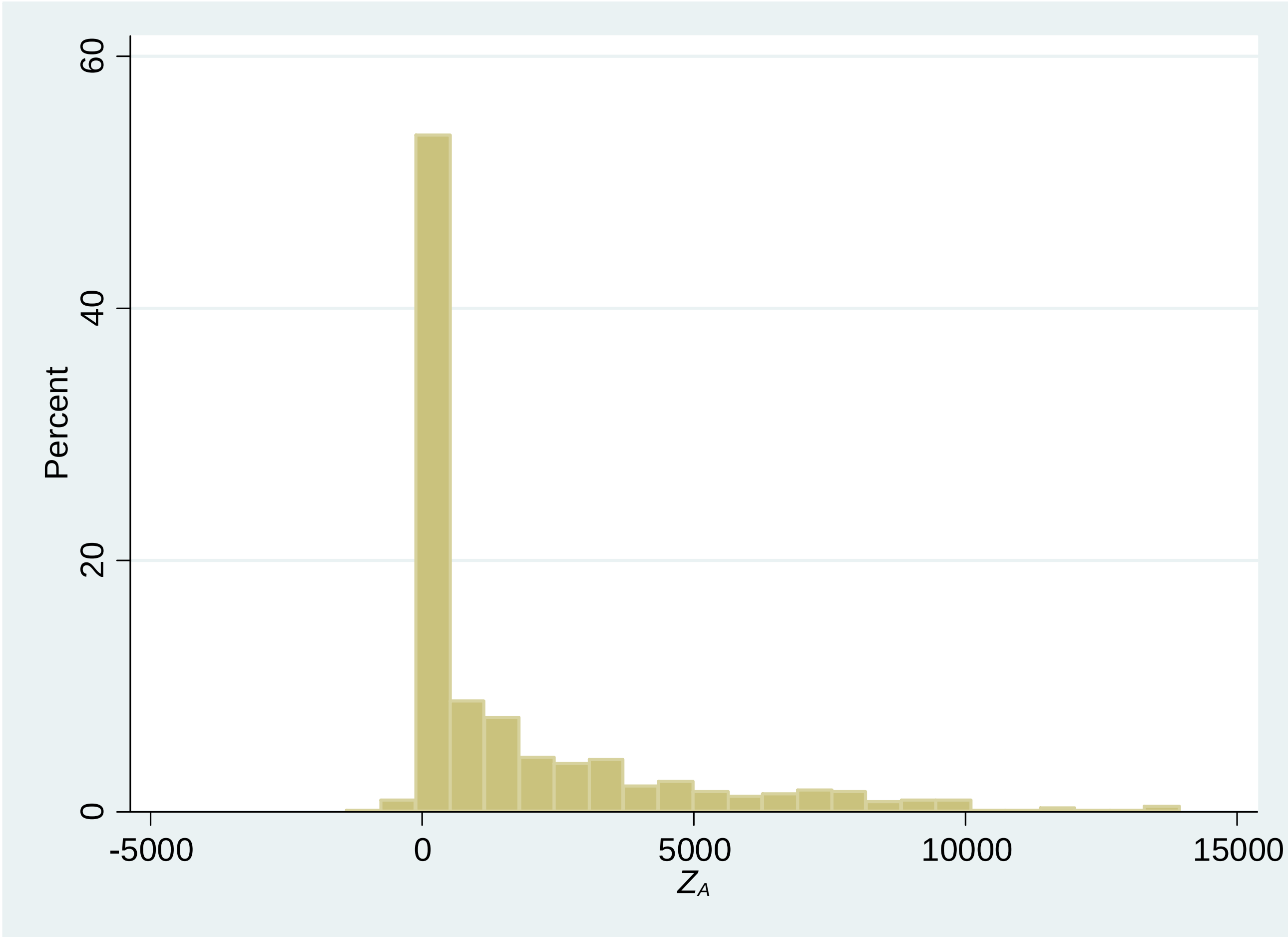
Calibration (3)

- Transition Probability Matrix of Z_A (Π)

$$\begin{pmatrix}
0.6676 & 0.2177 & 0.0458 & 0.0258 & 0.0174 & 0.0045 & 0.0083 & 0.0086 & 0.0043 & 0.0000 \\
0.2177 & 0.5400 & 0.2085 & 0.0252 & 0.0043 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0043 \\
0.0458 & 0.2085 & 0.5618 & 0.1541 & 0.0170 & 0.0000 & 0.0042 & 0.0000 & 0.0000 & 0.0086 \\
0.0258 & 0.0252 & 0.1541 & 0.5647 & 0.1804 & 0.0208 & 0.0165 & 0.0042 & 0.0000 & 0.0083 \\
0.0174 & 0.0043 & 0.0170 & 0.1804 & 0.5225 & 0.2331 & 0.0208 & 0.0000 & 0.0000 & 0.0045 \\
0.0045 & 0.0000 & 0.0000 & 0.0208 & 0.2331 & 0.5225 & 0.1804 & 0.0170 & 0.0043 & 0.0174 \\
0.0083 & 0.0000 & 0.0042 & 0.0165 & 0.0208 & 0.1804 & 0.5647 & 0.1541 & 0.0252 & 0.0258 \\
0.0086 & 0.0000 & 0.0000 & 0.0042 & 0.0000 & 0.0170 & 0.1541 & 0.5618 & 0.2085 & 0.0458 \\
0.0043 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0043 & 0.0252 & 0.2085 & 0.5400 & 0.2177 \\
0.0000 & 0.0043 & 0.0086 & 0.0083 & 0.0045 & 0.0174 & 0.0258 & 0.0458 & 0.2177 & 0.6676
\end{pmatrix}$$

- Depreciation Rate $\delta = 0.1054$

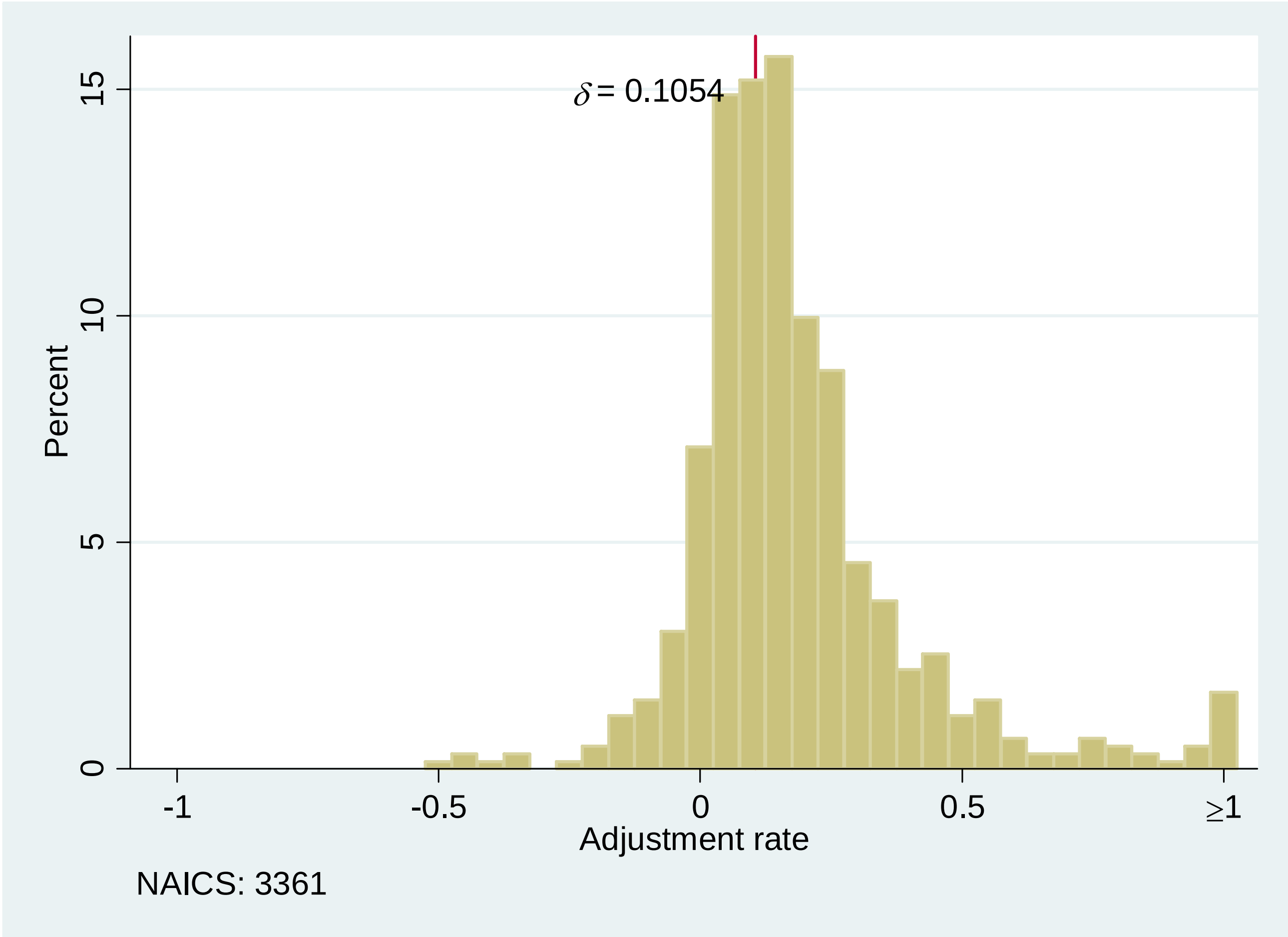
Estimated Z_A



623 Observations

Investment Rates (1)

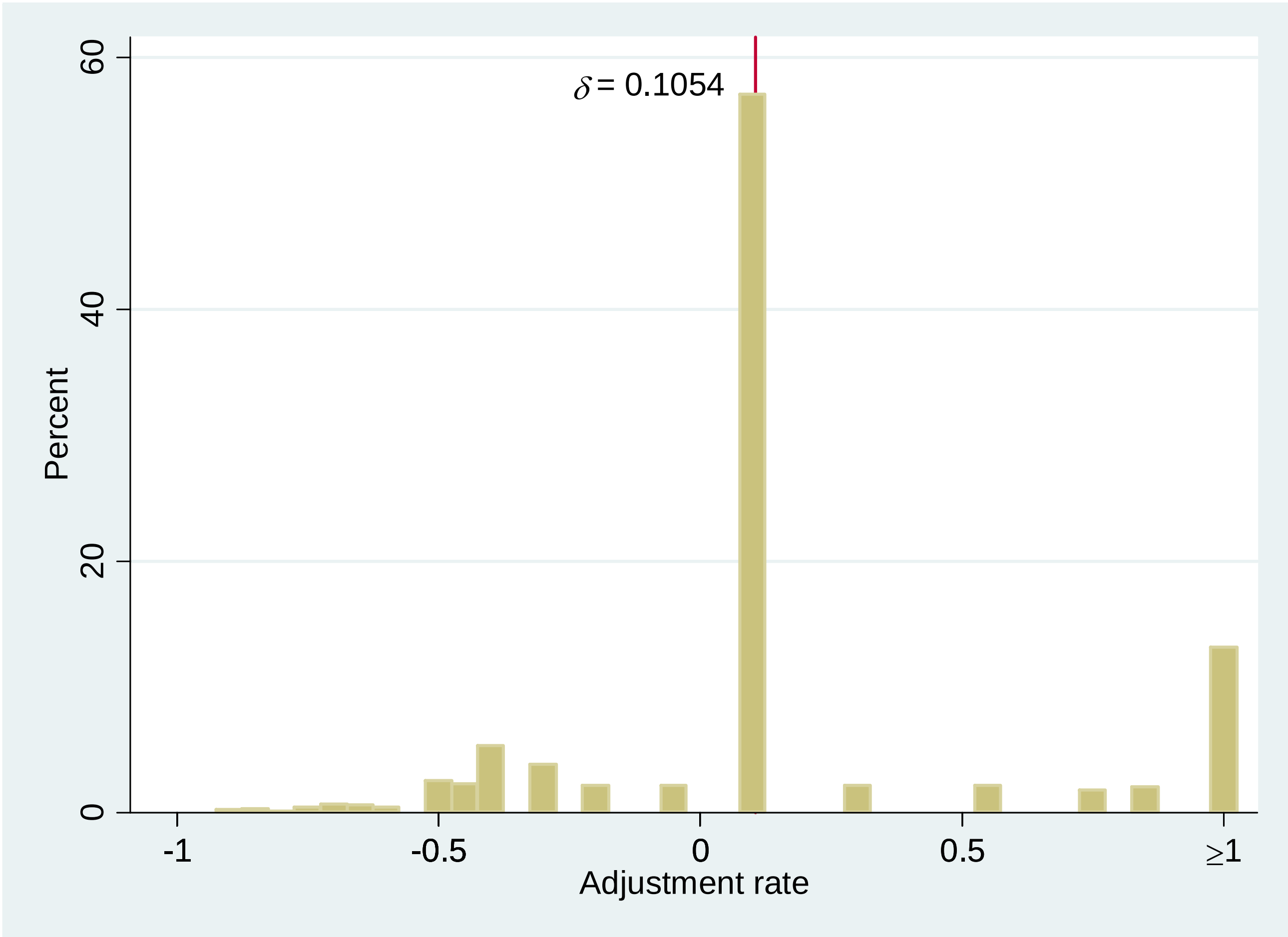
Observations



592 Observations, year 1980 to 2005

Investment Rates (2)

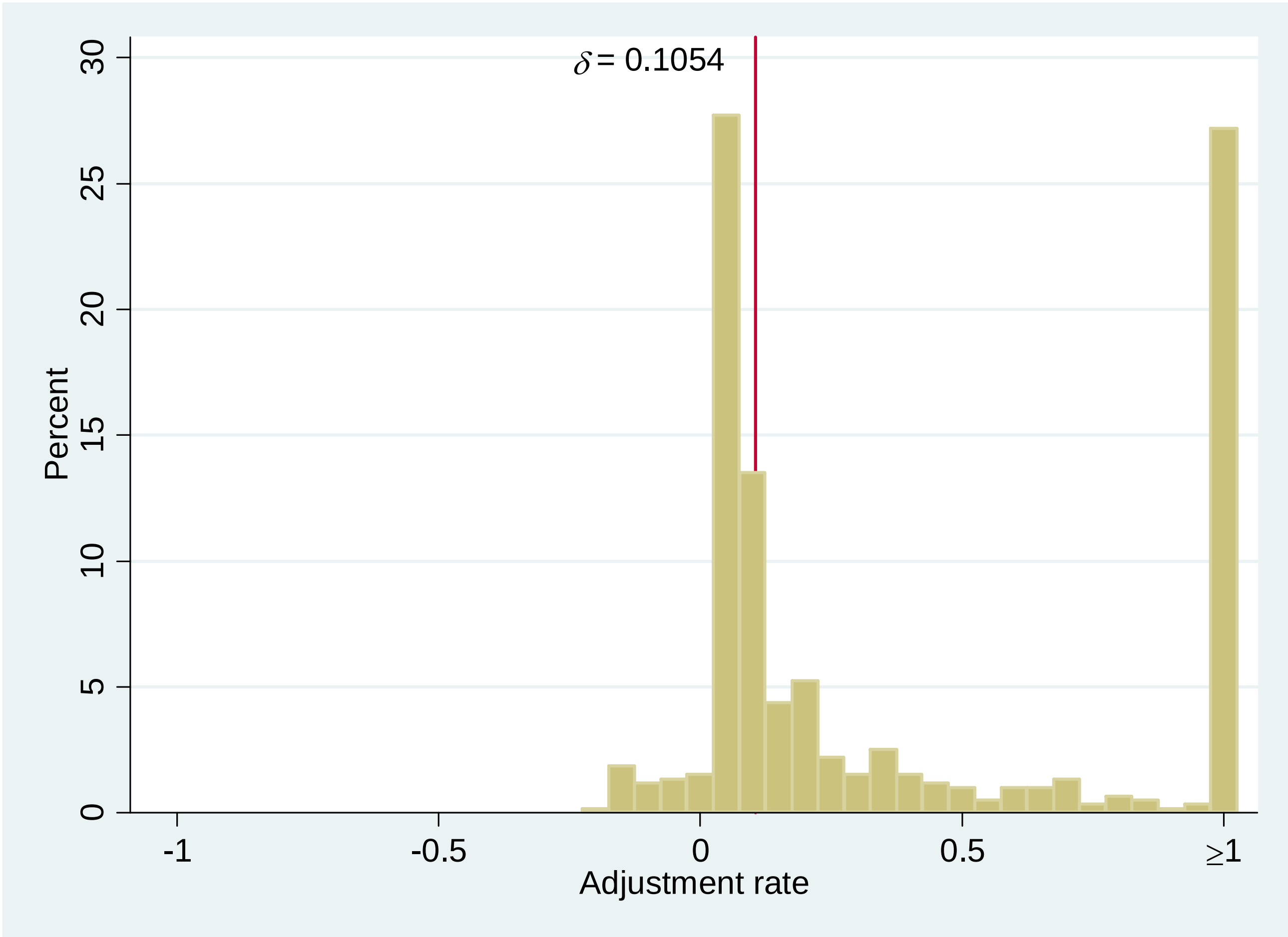
Predictions without Adjustment Costs



592 Predictions

Investment Rates (3)

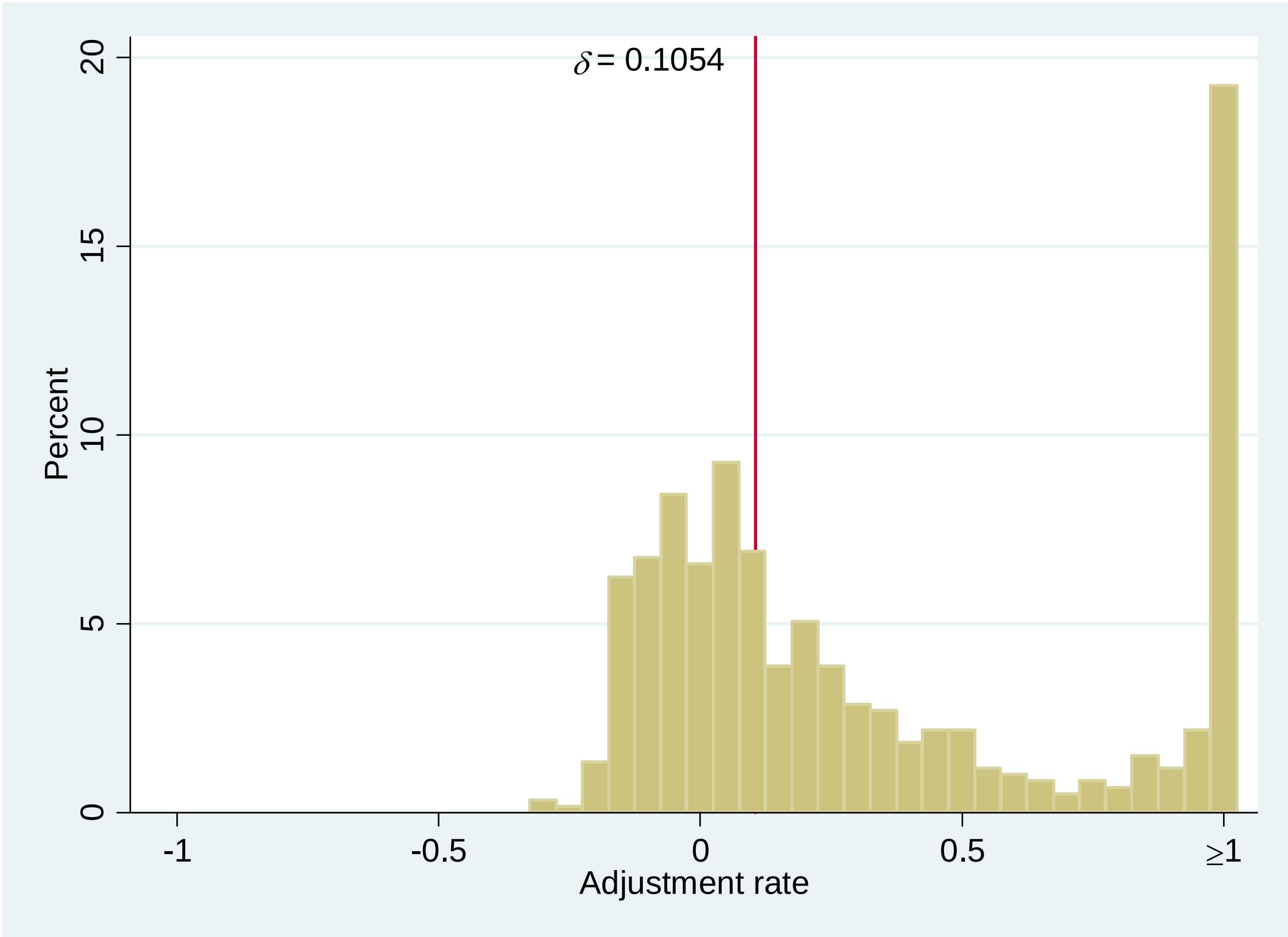
Predictions with I^2 and net investment



592 Predictions

Investment Rates (4)

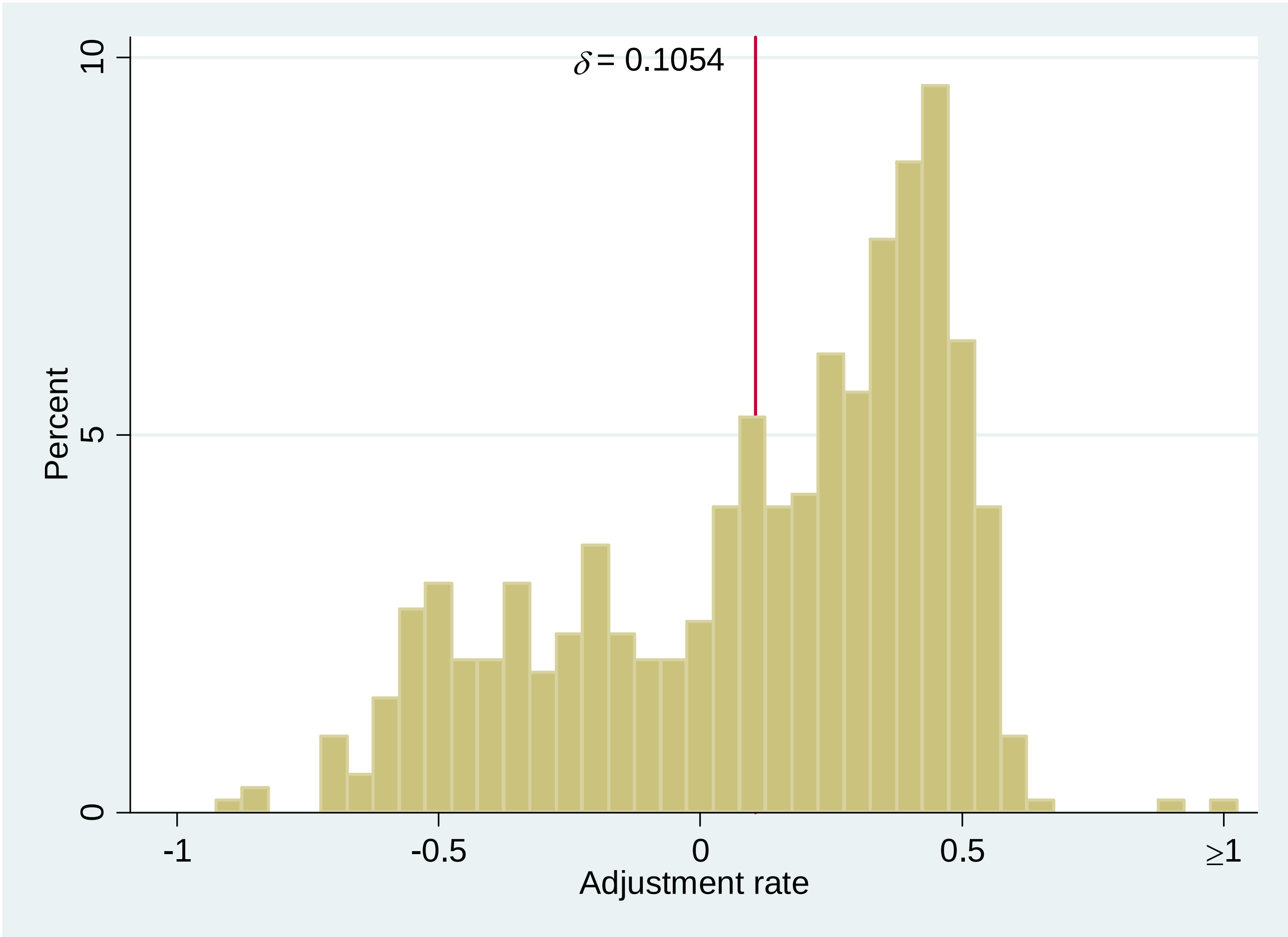
Predictions with I^2/K and net investment



592 Predictions

Investment Rates (5)

Predictions with $(I/K)^2$ and net investment

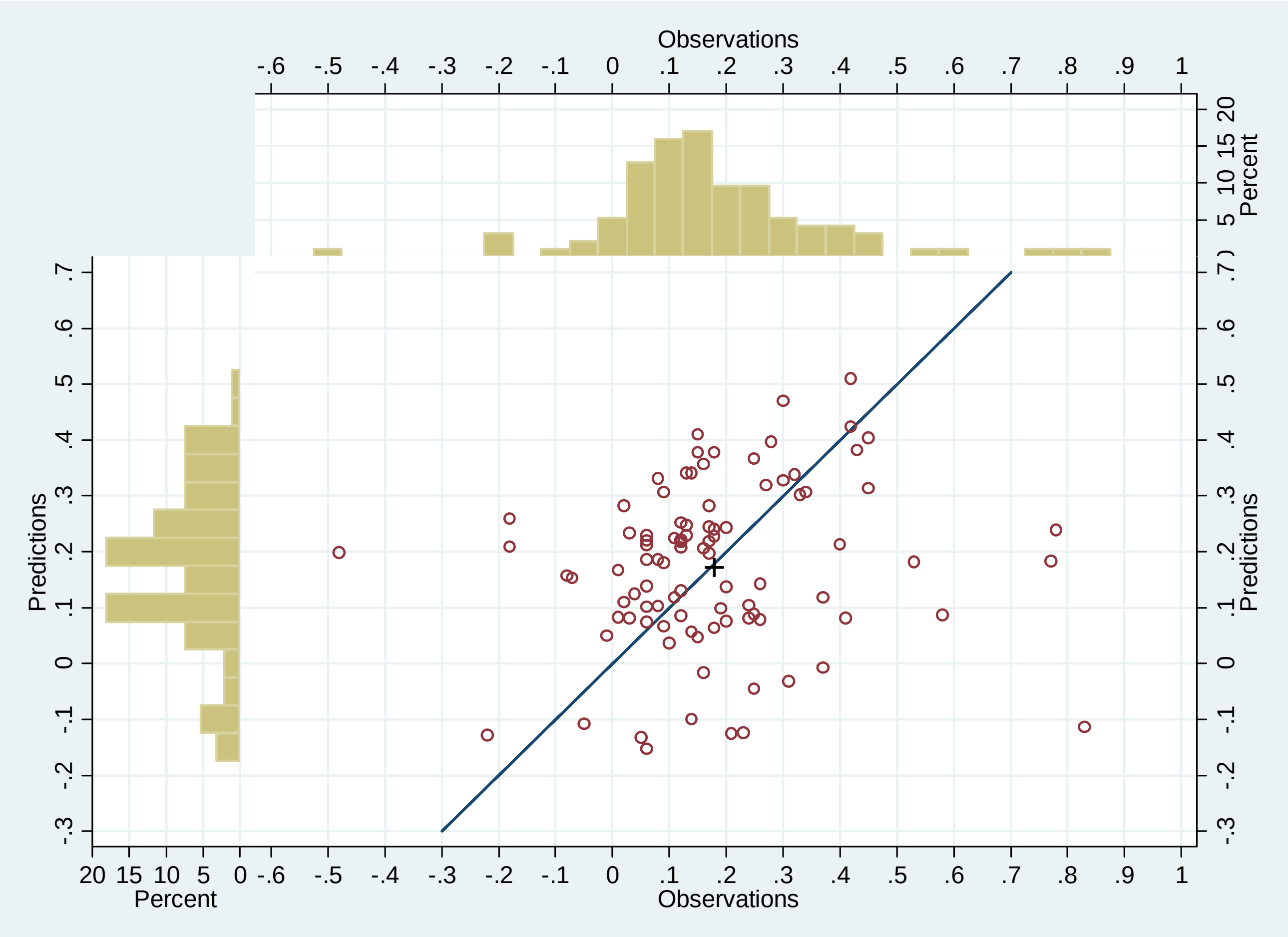


592 Predictions

Observations vs Predictions (1)

Predictions with I^2/K and net investment

<u>Medium firms</u> $K = 1{,}100 \sim 4{,}500$

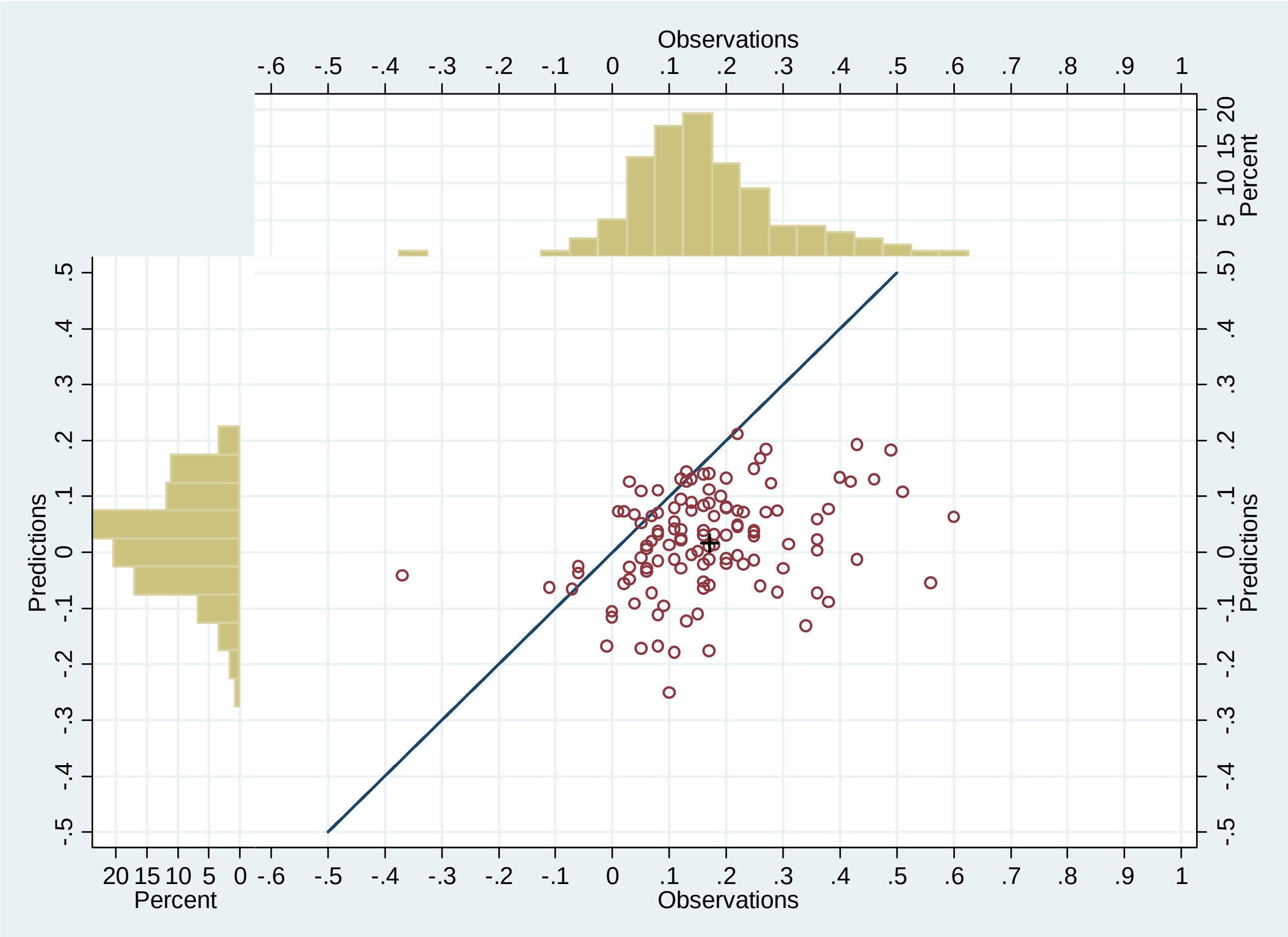


94 Observations

Observations vs Predictions (2)

Predictions with I^2/K and net investment

<u>Large firms</u> $K = 7{,}500 \sim 30{,}000$

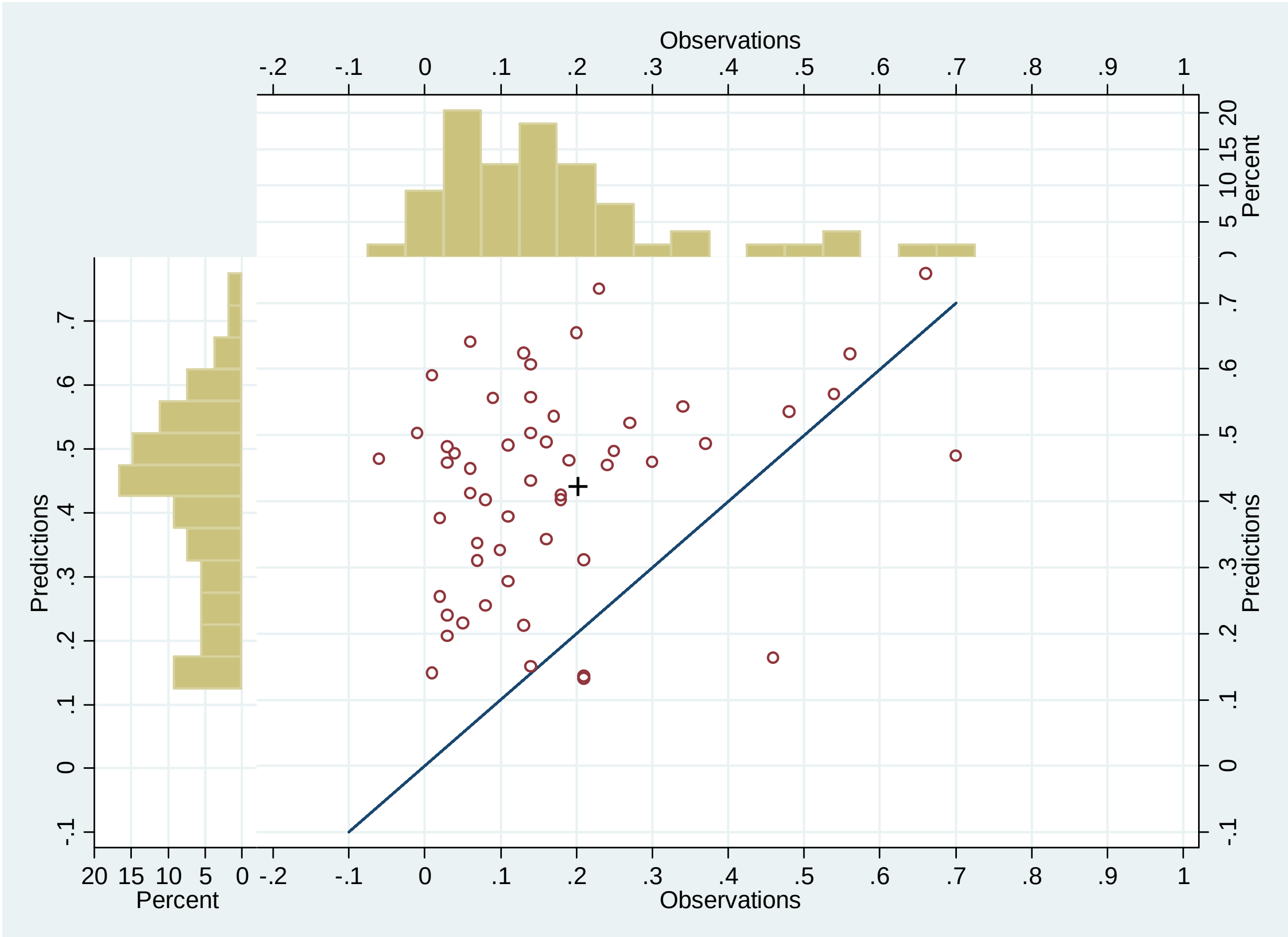


117 Observations

Observations vs Predictions (3)

Predictions with I^2/K and net investment

<u>Small firms</u> $K = 150{\sim}600$

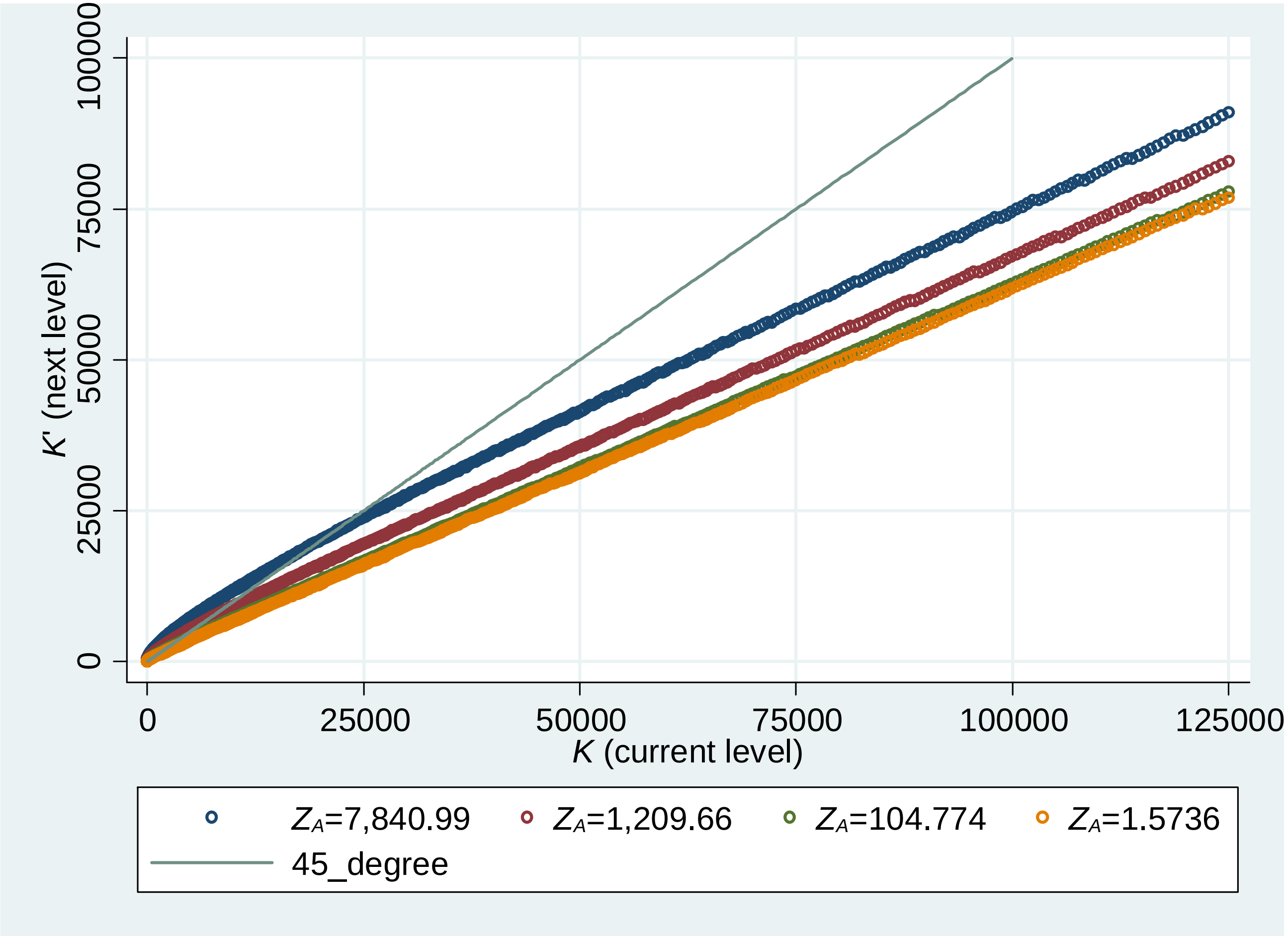


64 Observations

Policy Function (1)

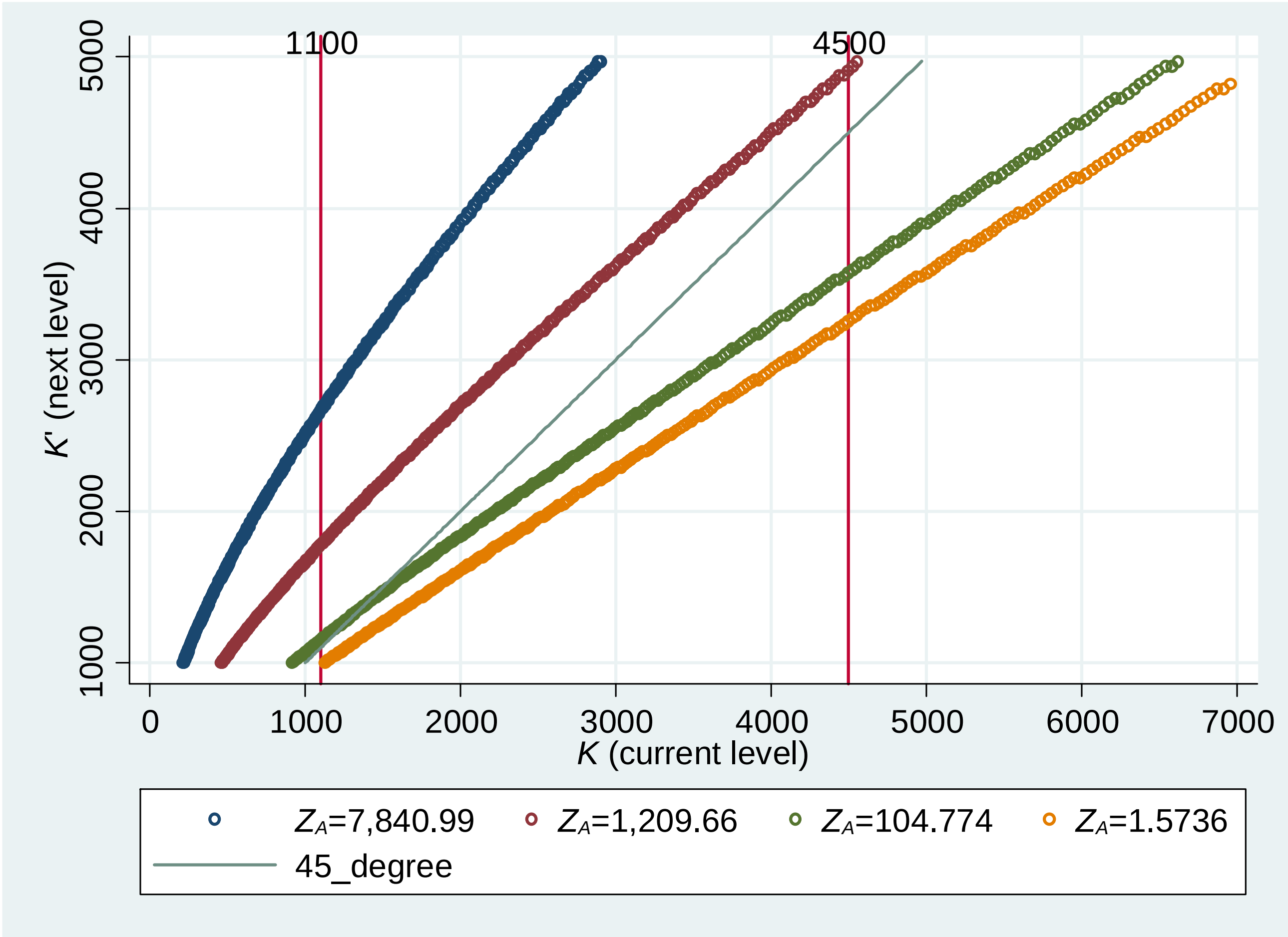
Predictions with I^2/K and net investment

Overview



Policy Function (2)

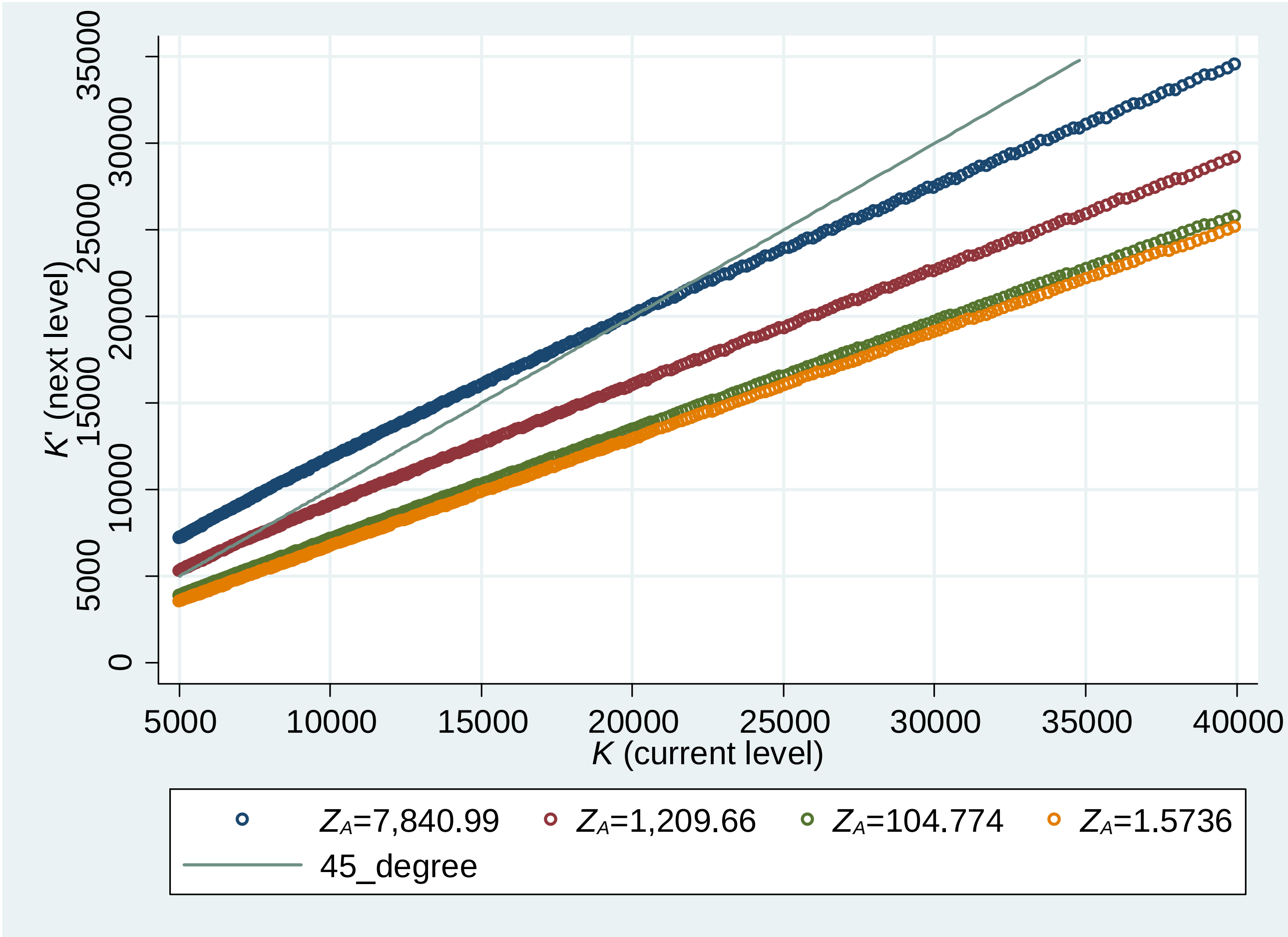
Predictions with I^2/K and net investment
<u>Medium firms</u> $K = 1{,}100 \sim 4{,}500$



Policy Function (3)

Predictions with I^2/K and net investment

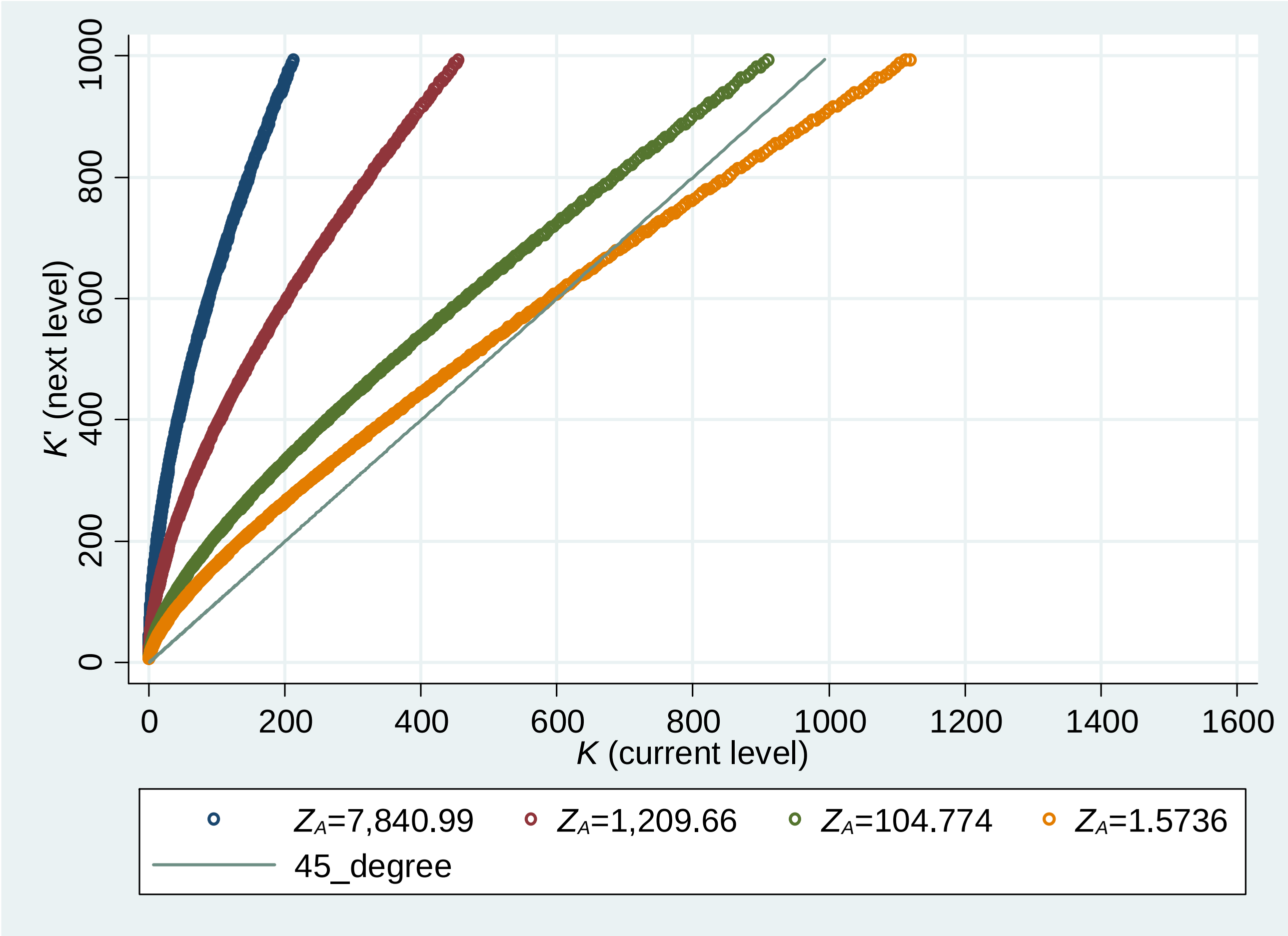
Large firms $K = 7{,}500 \sim 30{,}000$



Policy Function (4)

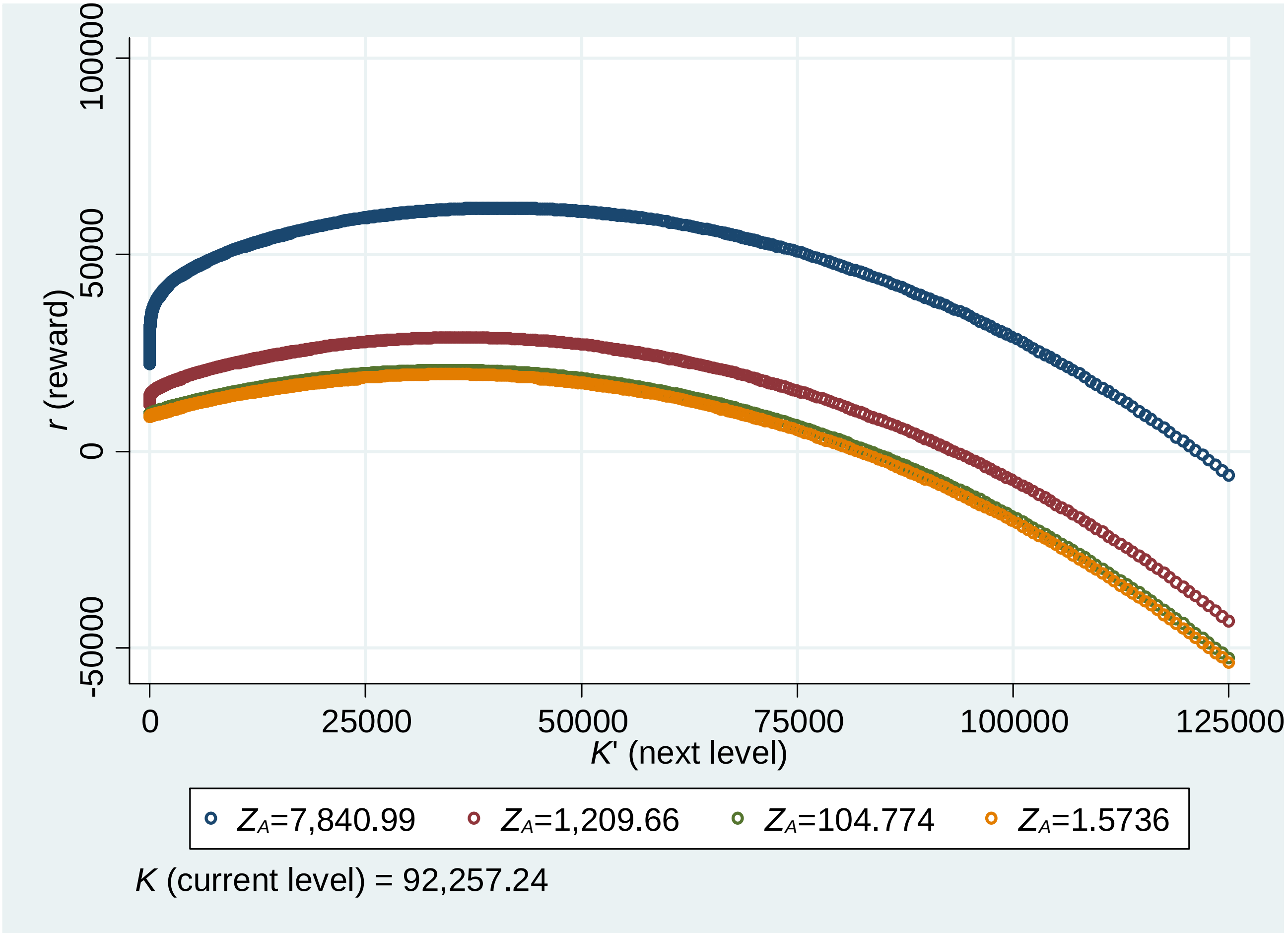
Predictions with I^2/K and net investment

<u>Small firms</u> $K = 150{\sim}600$



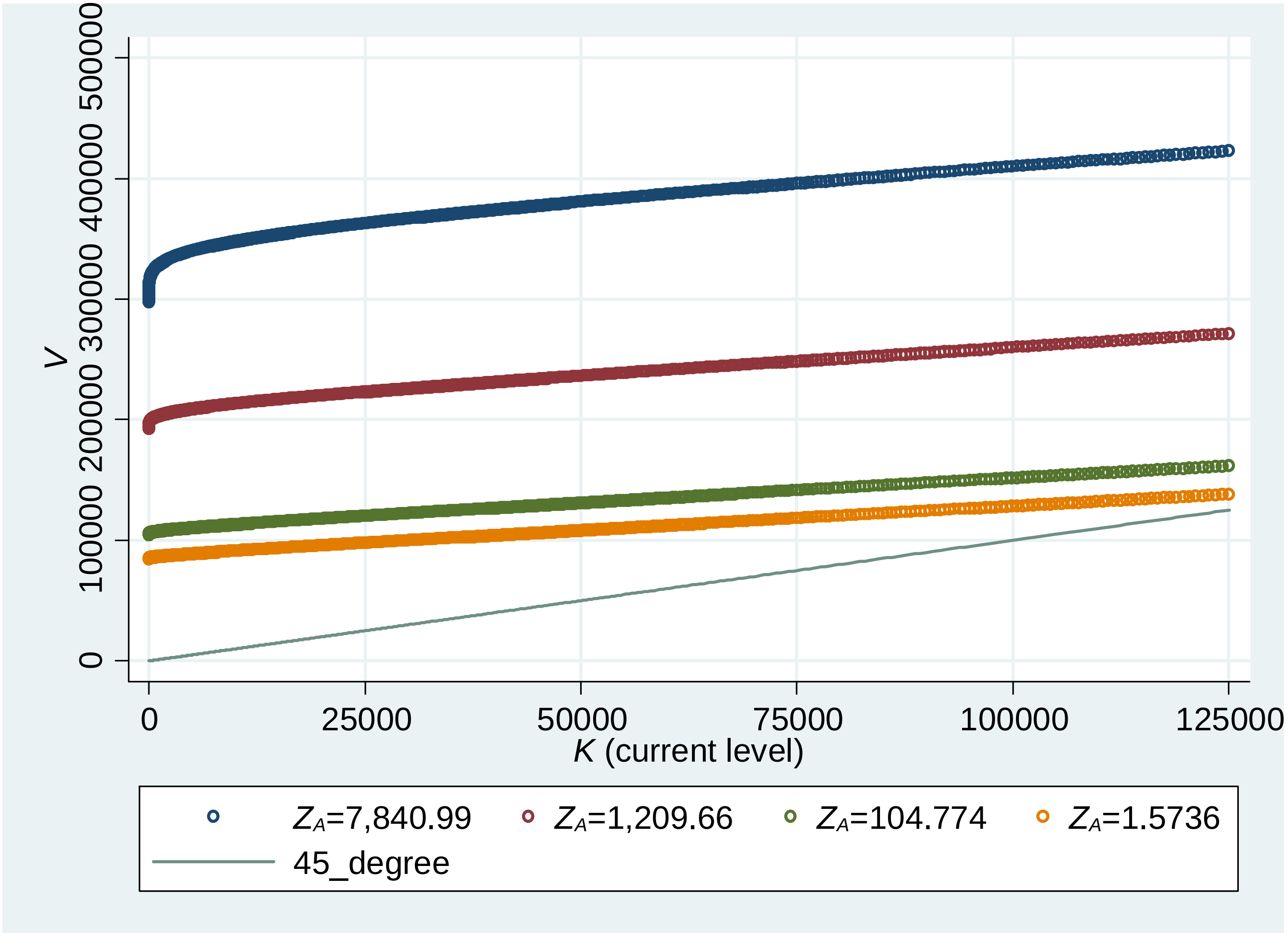
Reward Function

Predictions with I^2/K and net investment



Value Function

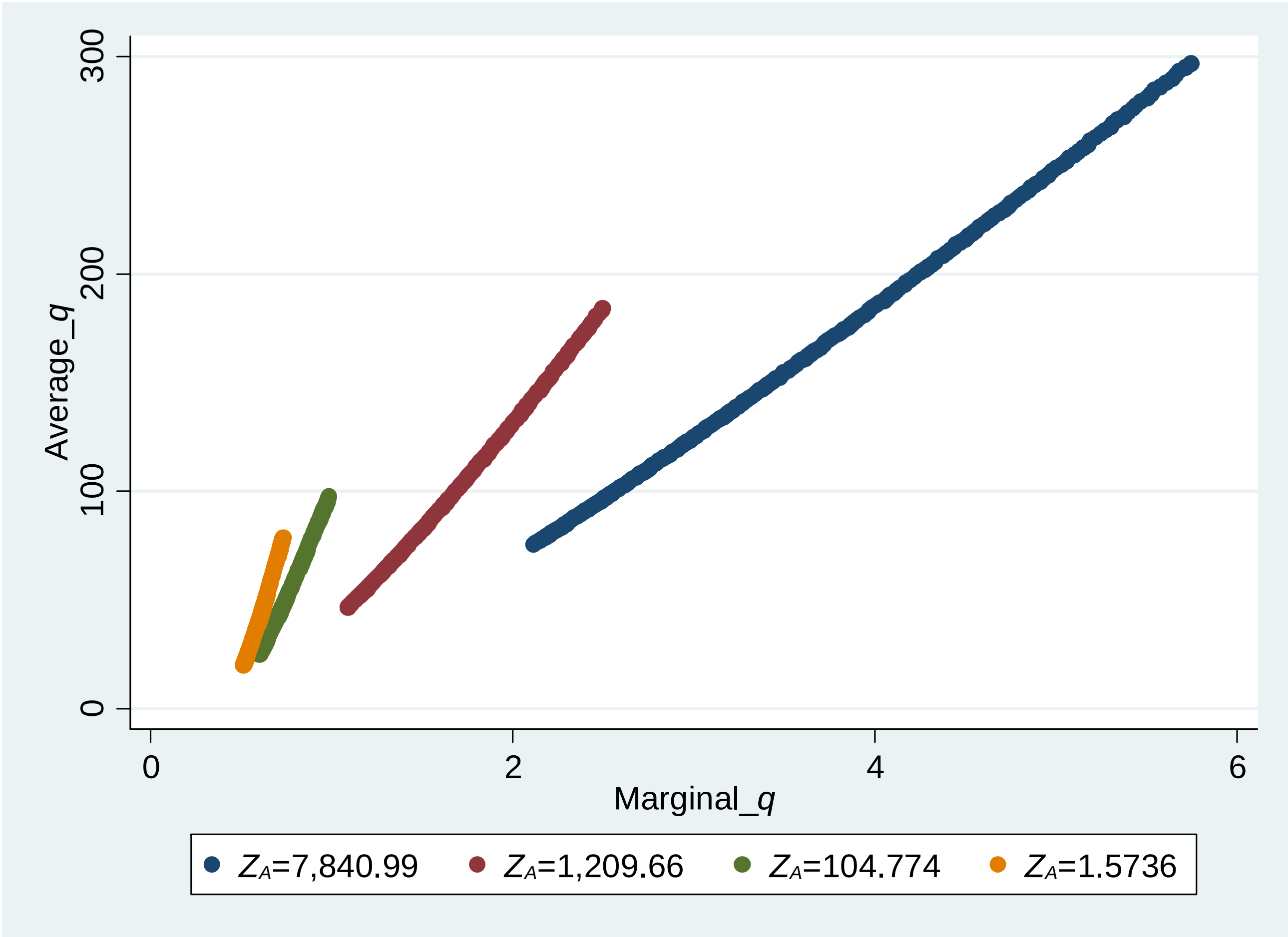
Predictions with I^2/K and net investment



Neoclassical Hayashi Model (1)

Predictions with I^2/K and net investment for medium firms

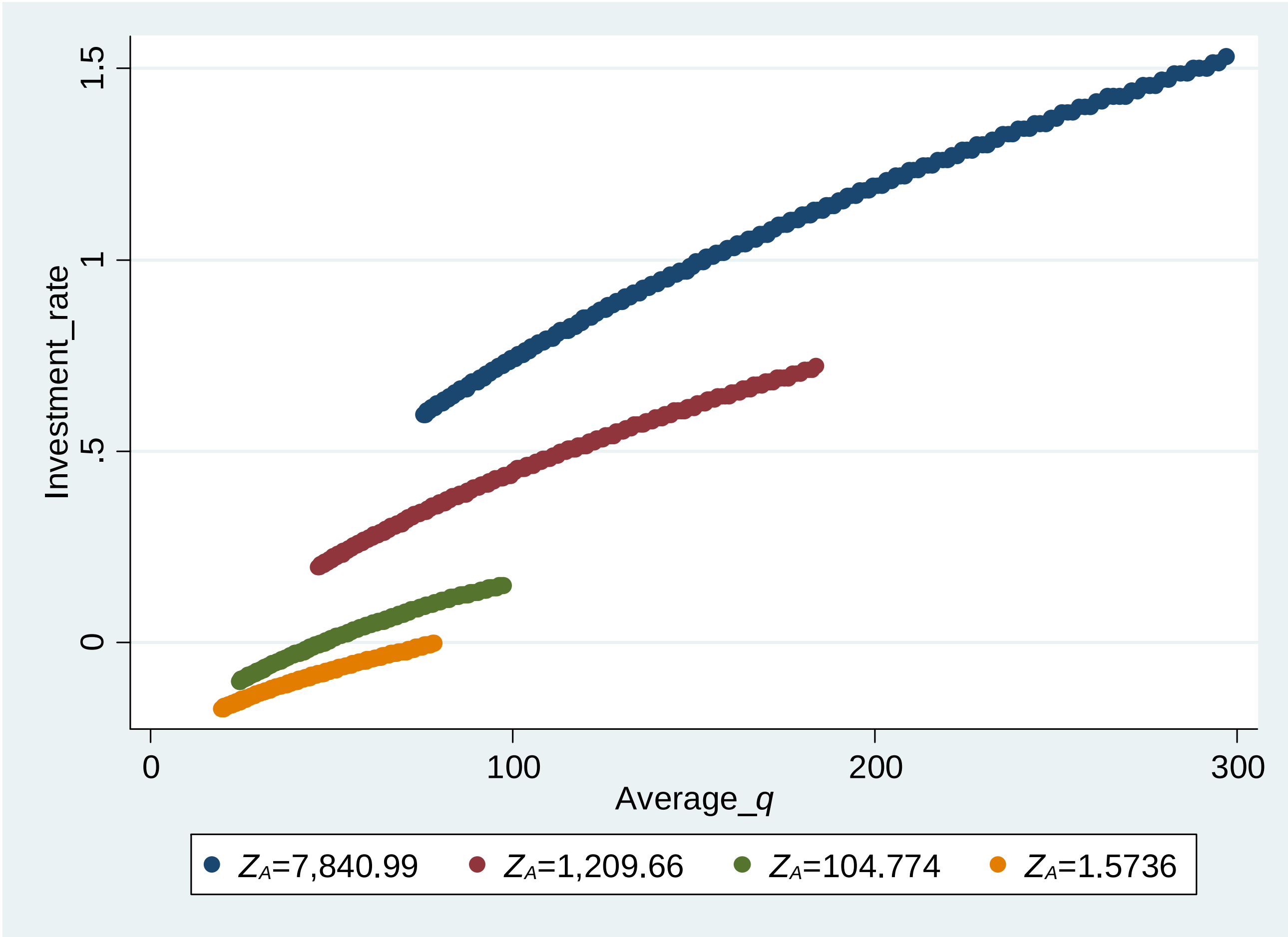
Marginal *q* vs Average *q*



Neoclassical Hayashi Model (2)

Predictions with I^2/K and net investment for medium firms

Investment function (Investment rate vs Average q)



Future Research

- Include the linear term
- Estimate the coefficients of the adjustment function by Nonlinear estimations
- Incorporate individual effects
- Effects of adjustment costs on capital accumulation and investment fluctuations
- Effects of volatility on capital adjustments
-