

Debt dilution and sovereign default risk*

Working Paper 10-08R

Juan Carlos Hatchondo Leonardo Martinez César Sosa Padilla

January 23, 2012

Abstract

We measure of the effects of debt dilution on sovereign default risk and show how these effects can be mitigated with contracts that specify debt-issuance-contingent obligations. First, we calibrate a baseline model à la Eaton and Gersovitz (1981) to match features of the data. In the baseline model bonds' values can be diluted. Second, we present a version of the baseline model in which the sovereign issues bonds with a covenant that compensates each bondholder for the difference between the post-issuance bond price and the counterfactual bond price that would have been observed absent debt issuances in the current period. This issuance-contingent payment makes the value of each bond independent from future debt issuances and thus, it eliminates debt dilution. We quantify the effects of dilution by comparing the simulations of the baseline and the modified model. We find that dilution accounts for 84% of the default risk in the baseline economy. Similar default risk reductions can be obtained with issuance-contingent payments that depend only on market prices. Given that defaults are ex-ante inefficient, eliminating dilution is welfare enhancing because it reduces the frequency of defaults.

JEL classification: F34, F41.

Keywords: Sovereign Default, Debt Dilution, Overborrowing, Debt Covenant, Long-term Debt, Endogenous Borrowing Constraints.

*For comments and suggestions, we thank Fernando Alvarez, Luis Brandao-Marques, Fernando Broner, Satyajit Chatterjee, Pablo D'Erasmus, Bora Durdu, Enrique Mendoza, Dirk Niepelt, Jorge Roldos, Diego Saravia, and seminar participants at the U.S. Board of Governors of the Federal Reserve System, the University of Virginia, the Federal Reserve Bank of Richmond, the Federal Reserve Bank of Atlanta, the Universidad Nacional de Tucumán, the 2009 Wegmans conference, the Pontificia Universidad Católica de Chile, the Central Bank of Chile, the 2010 Royal Economic Society Meeting, the 2010 Eastern Economic Association Meeting, the 2010 SED conference, the 2010 Berlin Conference on Sovereign Debt and Default, the 2010 and 2011 AEA meetings, the ECB, and the 2011 NBER International Finance and Macroeconomics Workshop. Remaining mistakes are our own. The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management, the Federal Reserve Bank of Richmond, or the Federal Reserve System.
E-mails: JuanCarlos.Hatchondo@rich.frb.org; leo14627@gmail.com; sosa@econ.umd.edu.

1 Introduction

This paper presents a measure of the effects of the debt dilution induced by governments' borrowing decisions alone and show how these effects can be mitigated with contracts that specify debt-issuance-contingent obligations. If governments could commit to not dilute the value of current bond issuances with future issuances, this would allow them to sell bonds at a higher price. Sovereign debt dilution may become a problem when governments do not have the ability to make such commitment.

Participants in various credit markets have made efforts to mitigate the dilution problem, which is suggestive of the relevance assigned to this issue. First, we observe debt claims with different seniority. This is clear in corporate debt and in collateralized loans to households. In contrast, for sovereign bonds, we are not aware of the use of differences in legal seniority.¹ Second, in some markets debt contracts include covenants intended to limit debt dilution (see Smith and Warner (1979) and Rodgers (1965) for a discussion of debt covenants in corporate debt markets). In sovereign debt contracts it is common to introduce a *pari passu* clause and many contracts also contain negative pledge clauses that prohibit future issuances of collateralized debt. These clauses intend to avoid making new bonds senior to previously issued bonds, but do not protect against dilution caused by future borrowing behavior.²

The weaker protection against sovereign debt dilution may be due in part to the weak enforcement in sovereign debt markets. This weak enforcement has led to several proposals to induce more orderly sovereign debt restructurings (see, for example, Bolton and Skeel (2005), Borensztein et al. (2004), G-10 (2002), IMF (2003), Krueger and Hagan (2005), and Paulus (2002)). For example, Bolton and Skeel (2005) argue for the importance of being able to grant seniority to debt issued while the country is negotiating with holders of debt in default, as ob-

¹It has been argued that loans from institutions such as the International Monetary Fund or the World Bank receive de facto seniority over loans from private agents (see, for example, Saravia (2010)).

²Sturzenegger and Zettelmeyer (2006) discuss that in the 2000 sovereign debt restructuring in Ecuador, the government exchanged defaulted bonds with new bonds that included a clause specifying that if there was a default within 10 years following the restructure agreement, the government had to extend new bonds to the holders of the restructured debt. Sturzenegger and Zettelmeyer (2006) argue that the "effect of this was to offer a (limited) protection of bond holders against the dilution of their claims by new debt holders in the event of default." However, the inclusion of such debt covenants is much more an exception than a rule.

served in corporate bankruptcy procedures. Borensztein et al. (2004) suggest changes in national and international laws that may facilitate the introduction of debt contracts that provide some protection against debt dilution. Overall, it seems clear that existing sovereign debt contracts do not eliminate the risk of debt dilution.

We contribute to the discussion of sovereign debt dilution by providing a measure of its effects through the lens of a baseline default framework à la Eaton and Gersovitz (1981).³ Formally, we analyze a small open economy that receives a stochastic endowment stream of a single tradable good. The government’s objective is to maximize the expected utility of private agents. Each period, the government makes two decisions. First, it decides whether to default on previously issued debt. Second, it decides how much to borrow. The government can borrow by issuing non-contingent long-duration bonds, as in Hatchondo and Martinez (2009). The cost of defaulting is represented by an endowment loss that is incurred in the default period.

We then propose a new approach for the study of the effects of debt dilution. We modify the baseline sovereign default model by considering the case in which the sovereign issues debt with a covenant that entitles existing bondholders to receive a compensation payment equal to the difference between the post-issuance bond price and the counterfactual bond price that would have been observed absent new debt issuances. With this issuance-contingent payment, bond values become independent from future issuances and thus, there is no dilution caused by borrowing decisions. We measure the effects of dilution by comparing simulations of the baseline model (with dilution) with the ones of the modified model (without dilution). We impose discipline to our quantitative exercise by calibrating the baseline model to match data from an economy facing default risk (Argentina before its 2001 default).

We find that, even without commitment to future repayment policies, if the sovereign eliminates debt dilution, the number of default per 100 years decreases from 5.6 (with dilution) to 0.9 (without dilution). That is, dilution accounts for 84% of the default risk in the simulations of

³This approach has been used in many recent studies. See, for instance, Aguiar and Gopinath (2006), Alfaro and Kanczuk (2009) Arellano (2008), Boz (2011), Cuadra et al. (forthcoming), D’Erasmus (2008), Durdu et al. (2010), Lizarazo (2005, 2006), and Yue (2010). These models share blueprints with the models used in studies of household bankruptcy—see, for example, Athreya et al. (2007), Chatterjee et al. (2007), Li and Sarte (2006), Livshits et al. (2008), and Sanchez (2010).

the baseline model. Reducing the default frequency is beneficial for welfare because defaulting is ex-ante inefficient. Thus, our exercise is indicative of the quantitative importance of dilution and supports the view that dilution should be a central issue in discussions of sovereign debt management and the international financial architecture (e.g., Borensztein et al. (2004)).

Eliminating dilution allows the government to choose debt levels that command a low default risk. With dilution, default risk is high even if the government chooses very low debt levels that almost certainly would not trigger a default—in the following period. This is the case because future governments would increase the debt level. As long as a government cannot control the choices of future governments, it cannot choose low default risk.

The government's ability to choose low default risk is reflected in its borrowing opportunities (i.e., the set of combinations of levels of debt and interest rates the government can choose from). Eliminating dilution shifts the set of government's borrowing opportunities. The equilibrium combinations of debt and interest rate levels in the simulations without dilution are not part of the government's choice set with dilution. For no-dilution equilibrium debt levels, the equilibrium interest rate would be about 400 basis points higher in the economy with dilution. Issuance-contingent payments weaken the government's incentives to issue debt and thus imply lower future issuance levels. For any debt level, the expectation of lower future issuance levels implies a lower default probability. This in turn allows the government to pay a lower interest rate.

The compensation scheme that eliminates dilution requires knowledge of the price at which bonds would trade in the absence of current-period debt issuances. In practice, it may be difficult to determine the value of that price, so we present results using two simple compensation schemes that depend on variables that may be easier to verify. In one, issuance-contingent payments are a predetermined fixed share of current issuance revenues. In the other one, issuance-contingent payments are a decreasing function of the post-issuance bond price. The default frequency is reduced by 69% with the first scheme and is reduced by 86% with the second scheme.

It should be emphasized that our findings are not based on the assumption that the government can commit to issuance-contingent payments while it cannot commit to other types of payments. Sovereign debt contracts often contain an acceleration clause and a cross-default clause (for example, see IMF (2002)). The first clause allows creditors to call the debt they hold

in case the government default on a payment. The cross-default clause states that a default in any government obligation constitute a default in the contract containing that clause. These clauses imply that in practice, when the government chooses to default on a payment it chooses to default on all its debt. The implementation of issuance-contingent payments only require the assumption that defaulting on issuance-contingent payments would trigger acceleration and cross-default clauses and, therefore, a default on all government debt.

The issuance-contingent payments studied in this paper resemble covenants commonly used in corporate debt contracts to transfer resources from debtors to creditors when credit quality deteriorates (for instance, because of an increase in indebtedness). For instance, Asquith et al. (2005) document such “interest-increasing performance pricing” and find lower interest rates for contracts with this pricing.

Issuance-contingent payments also resemble taxes used in previous studies for eliminating overborrowing by private debtors (see Bianchi (forthcoming) and the references therein). In these studies, borrowing by one agent increases other agents’ cost of borrowing and the probability of a crisis. Taxing private borrowing reduces the frequency of crises. In this paper, the borrowing by future governments increases the current government’s cost of borrowing and the default probability. Issuance-contingent payments “tax” borrowing by future governments and thus reduces default risk.

1.1 Studies on debt dilution

The possibility of dilution has received considerable attention in both academic and policy discussions. Several theoretical studies describe the benefits of eliminating debt dilution. For instance, Bizer and DeMarzo (1992) show how dilution may lead to equilibria with higher debt levels and higher interest rates implied by higher default probabilities. It has also been argued that dilution may lead to excessive issuance of short-term debt (Kletzer (1984)), or of debt that is hard to restructure after a default (Bolton and Jeanne (2009)), which in turn could increase the likelihood and/or severity of sovereign debt crisis.⁴ While these studies suggest that debt dilution may be

⁴Detragiache (1994), Sachs and Cohen (1982), and Niepelt (2008) discuss further inefficiencies raised by sovereign debt dilution. Borensztein et al. (2004) suggest changes in national and international laws that may

an important source of inefficiencies in debt markets, they do not quantify the effects of dilution. We contribute to the discussion of sovereign debt dilution by measuring its effects and showing how they can be mitigated with issuance-contingent payments.

The most common modeling approach for the study of debt dilution is to focus on the effect of seniority clauses.⁵ If existing debt is senior to new issuances, this mitigates the dilution problem. However, it is well known that seniority does not fully eliminate debt dilution if new issuances increase the default probability and the expected recovery rate—i.e., the fraction of the loan lenders expect to recover after a default—is less than 100% (see, for example, Bizer and DeMarzo (1992)). Therefore, in general, one cannot measure accurately the effects of dilution by comparing the equilibria with and without seniority. Furthermore, seniority clauses may not be useful to eliminate sovereign debt dilution in reality. It is not clear how one could force a sovereign that defaults to respect a seniority structure.

A second approach for the study of debt dilution is to compare the equilibria obtained with long-duration and one-period bonds.⁶ Intertemporal debt dilution only appears with long-duration bonds. With one-period bonds, when the government decides its current issuance level, the outstanding debt level is zero (either because the government honored its debt obligations at the beginning of the period or because it defaulted on them). Thus, the government cannot dilute the value of debt issued in previous periods. However, one-period bonds do not only eliminate dilution but may also increase rollover risk. We show that, in general, one cannot measure accurately the effects of dilution by comparing the equilibria with one-period and long-duration bonds: Lower default probabilities with one-period bonds are not only the result of the elimination of debt dilution but also the result of the lower debt levels chosen by the borrower to mitigate rollover risk. Furthermore, while eliminating dilution with issuance-contingent pay-

facilitate the introduction of debt contracts that provide some protection against debt dilution. Bolton and Skeel (2005) argue for the importance of being able to grant seniority to debt issued while the country is negotiating with holders of debt in default, as observed in corporate bankruptcy procedures. See also Eaton and Fernandez (1995), Tirole (2002), and UN (2004).

⁵For instance, Bi (2006) analyzes a model with one-quarter and two-quarter bonds and studies the effects of making earlier issuances senior to new issuances. She finds that this decreases the default frequency but increases the mean debt level (perhaps because the endogenous borrowing constraint in the model is relaxed by making earlier issuances less risky).

⁶Chatterjee and Eyigungor (forthcoming) and Hatchondo and Martinez (2009) show that in a sovereign default framework, equilibrium default risk is significantly higher with long-duration bonds than with one-period bonds.

ments increases welfare, replacing long-duration bonds by one-period bonds in the presence of rollover risk decreases welfare. This illustrates how one-period bonds are not an alternative to eliminate dilution in practice. We believe the issuance-contingent payments studied in this paper provide a more robust approach to study the effects of debt dilution and may also represent a more promising tool for dealing with the debt dilution problem in reality.

The rest of the article proceeds as follows. Section 2 introduces the model. Section 3 discusses the calibration. Section 4 presents the results. Section 5 concludes and discusses possible extensions of our analysis.

2 The model

We first discuss the baseline model with debt dilution and later introduce issuance-contingent payments that allows us to quantify the role of debt dilution.

2.1 The baseline environment

There is a single tradable good. The economy receives a stochastic endowment stream of this good y_t , where

$$\log(y_t) = (1 - \rho) \mu + \rho \log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$, and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

The government's objective is to maximize the present expected discounted value of future utility flows of the representative agent in the economy, namely

$$E \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right],$$

where E denotes the expectation operator, β denotes the subjective discount factor, and the utility function is assumed to display a constant coefficient of relative risk aversion denoted by γ . That is,

$$u(c) = \frac{c^{(1-\gamma)} - 1}{1 - \gamma}.$$

As in Hatchondo and Martinez (2009), we assume that a bond issued in period t promises an infinite stream of coupons, which decreases at a constant rate δ . In particular, a bond issued in period t promises to pay one unit of the good in period $t + 1$ and $(1 - \delta)^{s-1}$ units in period $t + s$, with $s \geq 2$.

Each period, the government makes two decisions. First, it decides whether to default. Second, it chooses the number of bonds that it purchases or issues in the current period.

As in previous studies of sovereign default, the cost of defaulting is not a function of the size of the default. Thus, as in Arellano and Ramanarayanan (2010), Chatterjee and Eyigungor (forthcoming), and Hatchondo and Martinez (2009), when the government defaults, it does so on all current and future debt obligations. This is consistent with the behavior of defaulting governments in reality. As mentioned in the introduction, sovereign debt contracts often contain acceleration and cross-default clauses. These clauses imply that after a default event, future debt obligations become current.⁷ Following previous studies, we also assume that the recovery rate for debt in default is zero.

Lenders are risk neutral and assign the value e^{-r} to payoffs received in the next period. Bonds are priced in a competitive market inhabited by a large number of identical lenders, which implies that bond prices are pinned down by a zero expected profit condition.

When the government defaults, it faces an income loss of $\phi(y)$ in the default period.⁸ Following Chatterjee and Eyigungor (forthcoming), we assume a quadratic loss function $\phi(y) = d_0y + d_1y^2$.

We will also show that our measurement of the quantitative effects of dilution on default risk is robust to assuming that there is a shock to the cost of borrowing that is not perfectly correlated with the sovereign's income.

The government cannot commit to future default and borrowing decisions. Thus, one may

⁷The type of acceleration clauses depend on the details of each bond contract and on the jurisdiction under which the bond was issued (see IMF (2002)). For instance, in some cases it is necessary that creditors holding a minimum percentage of the value of the bond issue request their debt to be accelerated for their future claims to become due and payable. In other cases, no such qualified majority is needed.

⁸As in Hatchondo and Martinez (2009) and Hatchondo et al. (2009), we do not assume that the government is excluded from capital markets after a default episode. Hatchondo et al. (2007) solve a baseline model of sovereign default with and without the exclusion punishment and show that eliminating this punishment only affects significantly the debt level generated by the model.

interpret this environment as a game in which the government making the default and borrowing decisions in period t is a player who takes as given the default and borrowing strategies of other players (governments) who will decide after t . We focus on Markov Perfect Equilibrium. That is, we assume that in each period, the government's equilibrium default and borrowing strategies depend only on payoff-relevant state variables. As discussed by Krusell and Smith (2003), there may be multiple Markov perfect equilibria in infinite-horizon economies. In order to avoid this problem, we solve for the equilibrium of the finite-horizon version of our economy, and we increase the number of periods of the finite-horizon economy until value functions and bond prices for the first and second periods of this economy are sufficiently close. We then use the first-period equilibrium functions as the infinite-horizon-economy equilibrium functions.

2.2 Recursive formulation of the baseline framework

Let b denote the number of outstanding coupon claims at the beginning of the current period, and b' denote the number of outstanding coupon claims at the beginning of the next period. A negative value of b implies that the government was a net issuer of bonds in the past. Let d denote the current-period default decision. We assume that d is equal to 1 if the government defaulted in the current period and is equal to 0 if it did not. The number of bonds issued by the government is given by $-[b' - (1 - d)(1 - \delta)b]$. Let V denote the government's value function at the beginning of a period, that is, before the default decision is made. Let \tilde{V} denote its value function after the default decision has been made. Let F denote the conditional cumulative distribution function of the next-period endowment y' . For any bond price function q the function V satisfies the following functional equation:

$$V(b, y) = \max_{d \in \{0,1\}} \{d\tilde{V}(1, b, y) + (1 - d)\tilde{V}(0, b, y)\}, \quad (1)$$

where

$$\tilde{V}(d, b, y) = \max_{b' \leq 0} \left\{ u(c) + \beta \int V(b', y') F(dy' | y) \right\}, \quad (2)$$

and

$$c = y - d\phi(y) + (1 - d)b - q(b', y) [b' - (1 - d)(1 - \delta)b]. \quad (3)$$

The bond price is given by the following functional equation:

$$\begin{aligned}
q(b', y) &= \int e^{-r} [1 - h(b', y')] F(dy' | y) \\
&\quad + (1 - \delta) \int e^{-r} [1 - h(b', y')] q(g(h(b', y'), b', y'), y') F(dy' | y), \quad (4)
\end{aligned}$$

where h and g denote the future default and borrowing rules that lenders expect the government to follow. The default rule h is equal to 1 if the government defaults, and is equal to 0 otherwise. The function g determines the number of coupons that will mature next period. The first term in the right-hand side of equation (4) equals the expected value of the next-period coupon payment promised in a bond. The second term in the right-hand side of equation (4) equals the expected value of all other future coupon payments, which is summarized by the expected price at which the bond could be sold in the next period.

Equations (1)-(4) illustrate that the government finds its optimal current default and borrowing decisions taking as given its future default and borrowing decision rules h and g . In equilibrium, the optimal default and borrowing rules that solve problems (1) and (2) must be equal to h and g for all possible values of the state variables.

Definition 1 *A Markov Perfect Equilibrium is characterized by*

1. a set of value functions \tilde{V} and V ,
2. a default rule h and a borrowing rule g ,
3. a bond price function q ,

such that:

(a) given h and g , V and \tilde{V} satisfy equations (1) and (2) when the government can trade bonds at q ;

(b) given h and g , the bond price function q is given by equation (4); and

(c) the default rule h and borrowing rule g solve the dynamic programming problem defined by equations (1) and (2) when the government can trade bonds at q .

2.3 A model without debt dilution

In this section, we propose a modification to the model presented in Section 2.1 that will allow us to study an economy without debt dilution and, in turn, to measure the effects of debt dilution. We eliminate debt dilution—caused by borrowing decisions—by introducing issuance-contingent payments to bondholders. When the sovereign issues debt, it pays to the holder of each existing bond the difference between the post-issuance bond price $q(b', y)$ and the counterfactual bond price one would have observed absent new issuances: $q((1 - \delta)b, y)$. This issuance-contingent payment ensures that when buying sovereign debt, lenders anticipate that the future value of their investment is independent of future issuances. Thus, this issuance-contingent payment eliminates dilution.

We assume that a default on issuance-contingent payments also triggers acceleration and cross-default clauses that make all government's debt obligations become current. With this assumption, the government cannot gain by selectively defaulting on issuance-contingent payments. If it were to do so, it would have to cancel all current and future debt obligations. But this would be equivalent to first buying back all its debt and then issuing new debt (in this case, there would be no issuance-contingent obligations because all bondholders would have been paid off at the beginning of the period). The next subsection presents the recursive formulation of the no-dilution framework without giving to the government the option to default only on issuance-contingent payments.

2.4 Recursive formulation of the framework without debt dilution

As before, let q denote the price function of sovereign bonds. Let $\tilde{b} \equiv (1 - \delta)b < 0$ denote the interim number of next-period coupon obligations. Suppose the government issues $\tilde{b} - b' > 0$ bonds. Issuance-contingent payments are given by $-\tilde{b}[q(\tilde{b}, y) - q(b', y)]$. As in Section 2.1, when the government wants to buy back its bonds, it does so at the secondary-market price $q(b', y)$.

Suppose the bond price is higher when the debt level is lower because the default probability is increasing with respect to the debt level (as is always the case in the numerical cases we considered). The equilibrium bond price is given by

$$\begin{aligned}
q(b', y) &= \int e^{-r} [1 - h(b', y')] F(dy' | y) \\
&\quad + (1 - \delta) \int e^{-r} [1 - h(b', y')] \max \left\{ 0, q(\tilde{b}, y') - q(g(h(b', y'), b', y'), y') \right\} F(dy' | y) \\
&\quad + (1 - \delta) \int e^{-r} [1 - h(b', y')] q(g(h(b', y'), b', y'), y') F(dy' | y). \tag{5}
\end{aligned}$$

The first term of the right-hand side of equation (5) represents the expected value of the next-period coupon payment. The second term represents the expected issuance-contingent payments. The third term represents the expected next-period value of a bond after the lender received the issuance-contingent payment. Note that, because of the issuance-contingent payments, the future value of a lender's investment may be affected by the income shock, a debt buyback, and a default, but not by new issuances. Thus, there is no debt dilution in this framework.

The government's budget constraint reads

$$c = y - d\phi(y) + (1 - d)b + q(b', y)(\tilde{b} - b') + \tilde{b} \max\{0, q(\tilde{b}, y) - q(b', y)\}. \tag{6}$$

The last term of the right-hand side of equation (6) represents the government's issuance-contingent payment. After replacing equations (3) and (4) by equations (5) and (6) in the dynamic programming problem described in Section 2.2, we obtain the problem without debt dilution.

3 Calibration

Table 1 presents the calibration. We assume that the representative agent in the sovereign economy has a coefficient of relative risk aversion of 2, which is within the range of accepted values in studies of business cycles. A period in the model refers to a quarter. The risk-free interest rate is set equal to 1%. The parameter values that govern the endowment process are

Borrower's risk aversion	σ	2
Interest rate	r	1%
Output autocorrelation coefficient	ρ	0.9
Standard deviation of innovations	σ_ϵ	2.7%
Mean log output	μ	$(-1/2)\sigma_\epsilon^2$
Duration	δ	0.0341
Discount factor	β	0.969
Default cost	d_0	-0.69
Default cost	d_1	1.01
Risk premium	α	4

Table 1: Parameter values.

chosen so as to mimic the behavior of GDP in Argentina from the fourth quarter of 1993 to the third quarter of 2001, as in Hatchondo et al. (2009). The parameterization of the output process is similar to the parameterization used in other studies that consider a longer sample period (see, for instance, Aguiar and Gopinath (2006)).

With $\delta = 3.41\%$, bonds have an average duration of 4.19 years in the simulations of the baseline model.⁹ Cruces et al. (2002) report that the average duration of Argentinean bonds included in the EMBI index was 4.13 years in 2000. This duration is not significantly different from what is observed in other emerging economies. Using a sample of 27 emerging economies, Cruces et al. (2002) find an average duration of 4.77 years, with a standard deviation of 1.52.

We calibrate the discount factor and the output cost (two parameter values) to target three moments: a mean spread of 7.4, a standard deviation of the spread of 2.5, a mean debt level of 28% of the mean quarterly output in the pre-default samples of our simulations (the exact

⁹We use the Macaulay definition of duration, which with the coupon structure in this paper is given by

$$D = \frac{1 + r^*}{\delta + r^*},$$

where r^* denotes the constant per-period yield delivered by the bond.

definition of these samples is presented in Section 4.1.¹⁰ The targets for the spread distribution are taken from the spread behavior in Argentina before its 2001 default (see Table 2). Regarding the debt level, for the period we study, Chatterjee and Eyigungor (forthcoming) target a mean level of unsecured sovereign debt of 70% of quarterly output. Since our model is a model of external debt and Sturzenegger and Zettelmeyer (2006) estimate that 60% of the debt Argentina defaulted on was held by residents, we choose to target a mean debt level that is roughly 40% of the value targeted by Chatterjee and Eyigungor (forthcoming).

4 Results

Following Hatchondo et al. (2010), we solve the models numerically using value function iteration and interpolation.¹¹ First, we show that debt dilution accounts for most of the default risk in the benchmark economy. Second, we present the welfare gains from eliminating dilution. Third, we discuss the robustness of our measurement of the effects of debt dilution. Fourth, we compare long-duration debt with issuance-contingent payments to one-period debt. Fifth, we compare the allocation without dilution with the allocation that the government could attain if it could trade a full range of one-period state contingent bonds. Finally, we discuss alternative issuance-contingent payments that may be easier to implement.

4.1 Dilution and default risk

This subsection measures the effects of debt dilution. In order to do so, it presents simulation results from the models with and without debt dilution. Table 2 reports moments in the data and in our simulations.¹² As in previous studies, we report results for pre-default simulation samples. The exception is the default frequency, in which case we consider all simulation periods. We

¹⁰The discount factor value we obtain is relatively low but higher than the ones assumed in previous studies (for instance, Aguiar and Gopinath (2006) assume $\beta = 0.8$). Low discount factors may be a result of political polarization in emerging economies (see Amador (2003) and Cuadra and Sapriza (2008)).

¹¹We use linear interpolation for endowment levels and spline interpolation for asset positions. The algorithm finds two value functions, $\tilde{V}(1, \cdot, \cdot)$ and $\tilde{V}(0, \cdot, \cdot)$. Convergence in the equilibrium price function q is also assured.

¹²The data for output and consumption were obtained from the Argentinean Finance Ministry. The spread before the first quarter of 1998 is taken from Neumeyer and Perri (2005), and from the EMBI Global after that.

simulate the model for a number of periods that allows us to extract 500 samples of 32 consecutive periods before a default. We focus on samples of 32 periods because we compare the artificial data generated by the model with Argentine data from the fourth quarter of 1993 to the third quarter of 2001.¹³ In order to facilitate the comparison of simulation results with the data, we only consider simulation sample paths in which the last default was declared at least two periods before the beginning of each sample.

	Data	Dilution	No dilution
Defaults per 100 years		5.59	0.91
Mean debt market value		0.21	0.20
Mean debt face value	0.28	0.30	0.21
$E(R_s)$	7.44	7.21	0.95
$\sigma(R_s)$	2.51	2.54	0.68
$\sigma(y)$	3.17	2.97	3.28
$\sigma(c)/\sigma(y)$	0.94	1.03	1.10
$\rho(c, y)$	0.97	1.00	0.99
$\rho(R_s, y)$	-0.65	-0.81	-0.63

Table 2: Business cycle statistics. The second column is computed using data from Argentina from 1993 to 2001. Other columns report the mean of the value of each moment in 500 simulation samples. Each sample consists of 32 periods before a default episode.

The moments reported in Table 2 are chosen so as to illustrate the ability of the model to replicate distinctive business cycle properties of emerging economies. These economies feature a high, volatile, countercyclical interest rate, and high consumption volatility. To compute the quarterly interest rate spread we first calculate the yield that makes the present value of future payments promised in a bond—which for the no-dilution case includes issuance-contingent

¹³The qualitative features of this data are also observed in other sample periods and in other emerging markets (see, for example, Aguiar and Gopinath (2007), Alvarez et al. (2011), Boz et al. (2008), Neumeyer and Perri (2005), and Uribe and Yue (2006)). The only exception is that in the data we consider, the volatility of consumption is slightly lower than the volatility of income, while emerging market economies tend to display a higher volatility of consumption relative to income.

payments—equal to the bond price. Then, we calculate the quarterly spread as the difference between this yield and the risk-free rate. The annualized spread (R_s) is four times the quarterly spread.¹⁴

The logarithm of income and consumption are denoted by y and c , respectively. The standard deviation of x is denoted by $\sigma(x)$ and is reported in percentage terms. The coefficient of correlation between x and z is denoted by $\rho(x, z)$. Moments are computed using detrended series. Trends are computed using the Hodrick-Prescott filter with a smoothing parameter of 1,600. Table 2 also reports the mean debt market value (computed as the mean b divided by $\delta + r^*$, where r^* is the mean equilibrium interest rate) and the mean debt face value (computed as the mean b divided by $\delta + r$).

Table 2 shows that the baseline model with dilution matches the data reasonably well. As in the data, in the simulations of the baseline model, consumption and income are highly correlated, and the consumption volatility is higher than the income volatility. The model also approximates reasonably well the moments used as targets (the mean debt level, and the mean and standard deviation of the spread). As far as the default frequency is concerned, estimating the default probability in the data is difficult. Using a sample of 68 countries between 1970 and 2010, Cruces and Trebesch (2011) find a frequency of 6.6 defaults every 100 years. Arellano (2008) targets a frequency of three defaults per 100 years because that is the number of defaults observed in Argentina during the last 100 years. In Section 4.3, we show that, when the risk aversion of lenders is calibrated to generate a yearly default frequency of 3%, the effects of debt dilution are

¹⁴Formally, let \mathcal{C} denotes the compensation per bond in the economy without dilution when the initial state is given by the vector (b, y) , i.e.,

$$\mathcal{C}(b, y) = \max \{q(b(1 - \delta)(1 - h(b, y)), y) - q(g(h(b, y), b, y), y), 0\}.$$

Let q^{DF} denote the price of a default-free bond that pays the coupon and compensation \mathcal{C} in every period.

$$q^{DF}(b, y, i) = 1 + \mathcal{C}(b, y) + e^{-i}(1 - \delta) \int [(1 + \mathcal{C}(g(h(b, y), b, y), y')) + q^{DF}(g(h(b, y), b, y), y'))] F(dy' | y),$$

where i denotes the constant rate at which future payments are discounted. The yield of a risky bond in state (b, y) is defined as the value i^* that satisfies

$$q^{DF}(b, y, i^*) = q(g(h(b, y), b, y), y).$$

The annualized interest rate spread is therefore defined as $R_s = 4(i^* - r)$.

similar to the ones reported using our baseline calibration.

What are the quantitative effects of debt dilution? Table 2 shows that debt dilution accounts for 84% of the default risk in the simulations of the baseline model. The number of default per 100 years decreases from 5.59 in the baseline to 0.91 in the model without debt dilution. Debt dilution also accounts for 87% of the spread paid by the sovereign. The standard deviation of the spread decreases from 2.54 with debt dilution to 0.68 without debt dilution. The mean face value of outstanding bonds declines 32%. Most of this decline is explained by the lower interest rate in the simulations of the model without debt dilution: The mean market value of outstanding bonds decreases only by 9%.

In order to shed light on how the debt dilution problem influences equilibrium allocations, Figure 1 presents the spread demanded by lenders as a function of the face value of next-period debt. This function defines the set of combinations of spreads and next-period debt levels from which the government can choose. The figure also presents the combination of spread levels and next-period debt chosen by the government when its initial debt level is the average level in the simulations of each case.

Figure 1 shows that a shift in the government's choice set plays an important role in accounting for the reduction in spreads implied by the elimination of debt dilution: Even for the same debt levels, spread levels are higher in the benchmark than in the no-dilution model. For the equilibrium debt levels without dilution, equilibrium spread levels would be about 400 basis points higher in the economy with dilution. In the model without dilution, issuance-contingent payments weaken the governments incentives to issue debt and thus imply lower future issuance levels. For any debt level, the expectation of lower future issuance levels implies a lower default probability. This in turn allows the government to pay a lower interest rate.

Figure 1 helps in understanding why the consumption volatility is higher in the economy without dilution. This is the case because, for low income levels, consumption is more sensitive to changes income in that economy. As illustrated in Figure 1, when income is low, issuance levels tend to be lower in the economy without dilution than in the benchmark with dilution (in the figure, issuance levels are represented by the horizontal distance between the dark dots and

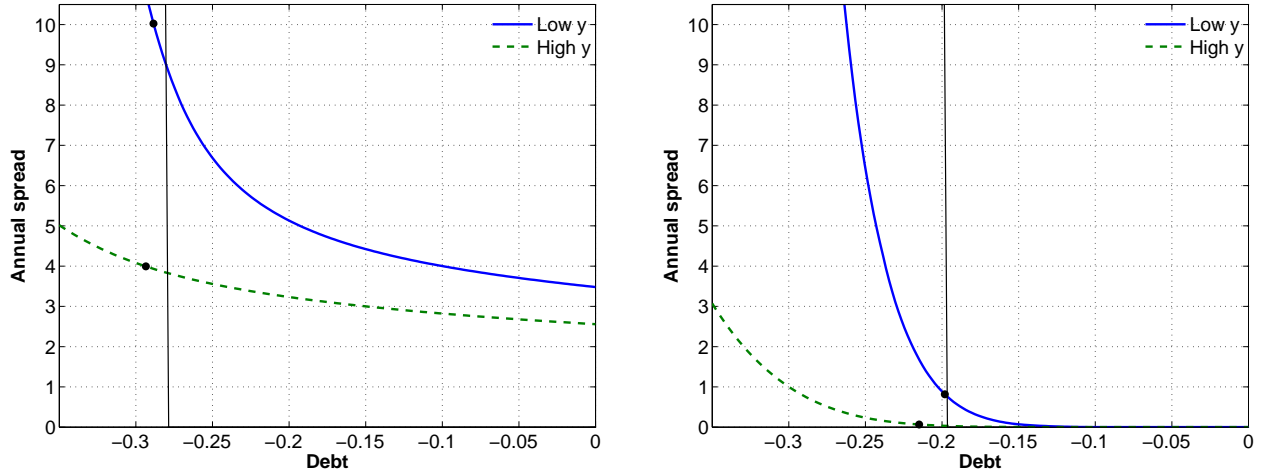


Figure 1: Menu of combinations of spreads and next-period debt levels ($\frac{b'}{\delta+r}$) from which the government can choose. The left panel corresponds to the baseline case. The right panel corresponds to the case without debt dilution. In each of these two cases, solid dots illustrate the optimal decision of a government that inherits a debt level equal to the average debt observed in our simulations for that case. Vertical lines mark the government's debt level before its issuance decision. The low (high) value of y corresponds to an endowment realization that is one standard deviation below (above) the unconditional mean.

the vertical solid line).¹⁵ Thus, the government is more effective in mitigating the effects of low income realizations on consumption in the benchmark with dilution.

4.2 Welfare gains from eliminating dilution

We next show that it is welfare enhancing to implement the issuance-contingent payments that eliminate debt dilution. Eliminating dilution reduces the frequency of defaults and with that, it reduces the inefficiencies caused by defaults. We measure welfare gains as the constant proportional change in consumption that would leave a consumer indifferent between continuing living in the benchmark economy with dilution and moving to an economy without dilution. Let V^{Dil} and $V^{\text{No dil}}$ denote the value functions in the benchmark economy and the economy without dilution, respectively. The welfare gain of moving from the benchmark economy to the economy

¹⁵Notice that, for the same debt level, the spread curves in Figure 1 are steeper when income is lower. This implies that in the economy without dilution, for the same issuance level, issuance-contingent payments are larger when income is lower.

without dilution is given by

$$\left(\frac{V^{\text{No dil}}(b, y)}{V^{\text{Dil}}(b, y)} \right)^{\left(\frac{1}{1-\sigma} \right)} - 1.$$

Figure 2 presents welfare gains from implementing the issuance-contingent payments that eliminate dilution. The figure considers two initial debt levels: zero, and the mean debt level in the simulations of the economy with dilution. The figure shows that for both cases there are welfare gains from eliminating dilution.

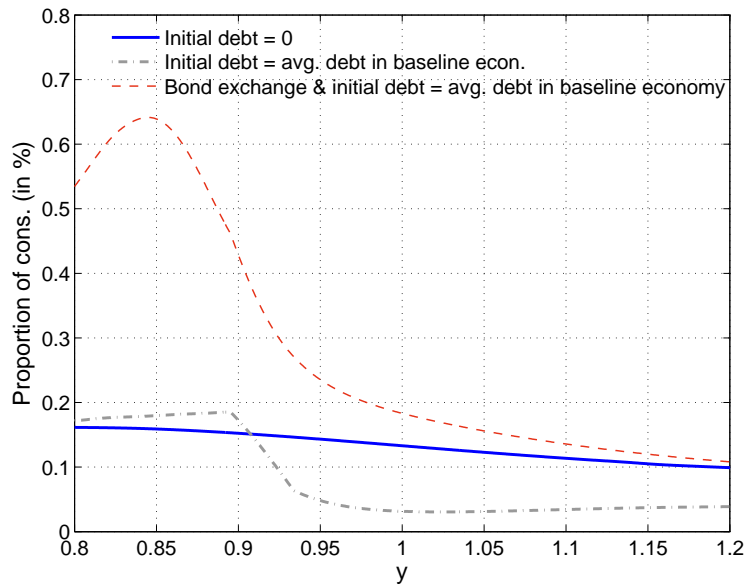


Figure 2: Consumption compensation (in percentage terms) that makes domestic agents indifferent between living in an economy with or without dilution. The figure was constructed assuming that the initial debt level is equal to zero or to the mean debt level in the simulations of the economy with dilution. A positive number means that agents prefer the economy without dilution.

In order to eliminate dilution in an economy with positive debt levels, the government promises issuance-contingent payments to holders of existing debt. This is costly for the government and explains why in Figure 2 welfare gains from eliminating dilution are lower when there is initial debt (except for lower income levels for which the government chooses to default).¹⁶

¹⁶Welfare gains are larger in a default period (when the government writes off all debt liabilities) than when the government enters de period without debt. After a default, the government wants to smooth out the income cost of defaulting. Thus, in a default period, the government has stronger incentives to borrow than when it enters

We also consider the case in which the government captures existing bondholders' capital gains from the introduction of issuance-contingent payments that eliminate debt dilution. We assume the government captures these gains through a debt exchange: The government makes a take-it-or-leave-it debt buyback offer with the promise that these issuance-contingent payments will be implemented only if this offer is accepted. Thus, the government offers bondholders to buyback previously issued bonds at the price that would have been observed if issuance-contingent payments were never going to be implemented. That price is lower than the no-dilution price at which the government would be able to issue debt after implementing issuance-contingent payments. By assuming that the government makes a take-it-or-leave-it offer, we focus on the extreme case in which it reaps all capital gains. The case in which issuance-contingent payments are introduced without a debt exchange constitutes the other extreme case in which bondholders enjoy all these gains. Figure 2 shows that welfare gains from eliminating dilution are positive for both cases.

It should be mentioned that one may want to take our measure of the welfare gain with a grain of salt. In particular, one could argue that our measure is too low. The welfare gain is increasing in the level of debt for which the risk premium is paid. We chose to calibrate our model to a low debt level to resemble the level of sovereign defaultable debt held by foreigners. However, eliminating debt dilution is also likely to reduce the risk premium paid for other government debt, and even private debt (sovereign spreads are likely to influence private spreads; see, for example, Mendoza and Yue (forthcoming)). Furthermore, several studies find evidence of a significant effect of interest rates on productivity (through the allocation of factors of production), and of a significant role of interest rate fluctuations in the amplification of shocks (see, for example, Mendoza and Yue (forthcoming), Neumeyer and Perri (2005), and Uribe and Yue (2006)). Since there is no production in our setup, we cannot capture productivity gains from reducing the level and volatility of interest rates.

the period without debt. This explains why the improvement in borrowing terms implied by the elimination of dilution is more valuable in default periods than when the government enters the period without debt.

4.3 Robustness

In this subsection, we show that our finding of a strong effect of debt dilution on sovereign default risk is robust to changes in the benchmark economy we consider. We focus on three changes to our benchmark. First, we modify the cost of defaulting to allow for a higher debt stock. Second, we assume that lenders are risk averse. This case allow us to calibrate the lenders's risk aversion to match the default frequency that is used as a target in previous quantitative studies. Third, we introduce sudden-stop shocks into the model (while we still assume risk-averse lenders). These shocks have received a lot of attention in the international macroeconomics literature (see Durdu et al. (2009) and the references therein) but have been mostly ignored in the quantitative sovereign default literature (Chatterjee and Eyigungor (forthcoming) is a noticeable exception). We show that eliminating dilution leads to a significant reduction in the default frequency in all these cases and that it is beneficial for welfare.

We allow for a higher debt stock by introducing a larger cost of defaulting: after a default the economy is banned from international capital markets for a stochastic number of periods and it losses a fraction of its output in every period it remains excluded. The probability of reentry to capital markets is constant over time. This is the same cost of defaulting considered in Aguiar and Gopinath (2006) and Arellano (2008). We assume that the probability of reentry equals 0.282, which is the value assumed in Arellano (2008). The only parameters that change compared to the benchmark parameterization are the ones that determine the output loss after a default. We set $d_0 = -0.7043$ and $d_1 = 0.9236$, which imply a debt level of 78 percent of quarterly output, a value slightly higher than the debt level targeted in Chatterjee and Eyigungor (forthcoming).

In order to introduce risk-averse lenders, we follow Arellano and Ramanarayanan (2010), we assume that the price of sovereign bonds satisfies a no arbitrage condition with stochastic discount factor $M(y', y) = e^{-r - \alpha \epsilon' - 0.5 \alpha^2 \sigma_\epsilon^2}$. This allows us to introduce risk premium into sovereign bond prices. Several studies document that the risk premium is an important component of sovereign spreads and that a significant fraction of the spread volatility in the data is accounted for by the volatility in the risk premium (see, for example, Borri and Verdelhan (2009), Broner et al. (2007), Longstaff et al. (2007), and González-Rozada and Levy Yeyati (2008)).

The previous discount factor is a special case of the discrete-time version of the Vasicek one-factor model of the term structure (see Vasicek (1977) and Backus et al. (1998)). With our formulation, the risk premium is determined by the income shock in the borrowing economy. It may be more natural to assume that the lenders' valuation of future payments is not perfectly correlated with the sovereign's income. However, the advantage of our formulation is that it avoids introducing additional state variables to the model.

The second robustness exercise shows that our measurement of the quantitative effect of dilution on default risk is robust to assuming that there is a shock to the cost of borrowing that is not perfectly correlated with the sovereign's income. We do so in a stark way by introducing sudden-stop shocks, i.e., we add an stochastic shock s such that when $s = 1$ ($s = 0$) the government can (cannot) issue debt—the government can always buy back previously issued debt. We denote by π_1 (π_0) the probability of $s_{t+1} = s_t$ conditional on $s_t = 1$ ($s_t = 0$). Thus, the value functions with sudden stops are given by

$$V(b, y, s) = \max_{d \in \{0,1\}} \{d\tilde{V}(1, b, y, s) + (1-d)\tilde{V}(0, b, y, s)\},$$

$$\tilde{V}(d, b, y, 1) = \max_{b' \leq 0} \left\{ u(c) + \beta \int [\pi_1 V(b', y', 1) + (1 - \pi_1) V(b', y', 0)] F(dy' | y) \right\},$$

and

$$\tilde{V}(d, b, y, 0) = \max_{b(1-\delta) \leq b' \leq 0} \left\{ u(c) + \beta \int [\pi_0 V(b', y', 0) + (1 - \pi_0) V(b', y', 1)] F(dy' | y) \right\}.$$

We calibrate π_1 and π_0 assuming that there is a 5.5% unconditional probability of observing $s = 0$ and sudden-stop periods last for one year on average (using annual data for a sample of developing countries from 1980 to 2003, Eichengreen et al. (2008) estimate a 5.5% probability of observing a sudden stop).

Table 3 presents simulation results with and without dilution for an economy with risk-averse lenders and for an economy with sudden-stop shocks. Table shows that a strong effect of debt dilution on sovereign default risk remains when we assume risk-averse lenders or sudden stops. The effects of dilution on other variables are also very similar across the different exercises.

Eliminating dilution is beneficial for welfare in all the cases studies and, as expected, the welfare gain is larger in the economy with a larger stock of debt: the distortions caused by defaults are larger in that economy.

	Higher debt		Risk-averse lenders		Sudden Stops	
	dilution	no dilution	dilution	no dilution	dilution	no dilution
Defaults per 100 years	4.52	1.02	3.10	0.42	5.07	0.86
Mean debt market value	0.55	0.56	0.20	0.18	0.23	0.19
Mean debt face value	0.78	0.59	0.28	0.19	0.32	0.20
$E(R_s)$	7.12	1.11	7.04	0.99	7.64	0.52
$\sigma(R_s)$	2.72	0.75	2.19	0.71	3.05	0.65
$\sigma(y)$	2.98	3.22	2.03	3.36	3.07	3.25
$\sigma(c)/\sigma(y)$	1.08	1.26	1.04	1.08	1.03	1.11
$\rho(c, y)$	0.99	0.98	1.00	0.99	0.99	0.99
$\rho(R_s, y)$	-0.79	-0.65	-0.82	-0.63	-0.80	-0.62
Welfare gain (% of cons.)		0.33		0.13		0.12

Table 3: Robustness exercises.

4.4 Issuance-contingent payments vs. one-period bonds

In this subsection, we show that in general one cannot measure the effects of debt dilution by comparing equilibria with long-duration and one-period bonds. Compared with long-duration bonds, one-period bonds do not only eliminate dilution but also increase rollover risk. With one-period bonds, when the government issues debt, the outstanding debt level is zero (either because the government honored its debt at the beginning of the period or because it defaulted on it) and, thus, the government cannot dilute the value of debt issued in previous periods. But with one-period bonds the government is also asked to payback all of its debt every period, which increases its exposure to rollover risk.

Table 4 presents simulation results for two models without dilution: Our model with issuance-contingent payments, and a one-period bond model (i.e., a model with $\delta = 1$). The table presents results for both the baseline and the sudden-stop versions of the model. Table 4 shows that simulation results obtained with one-period bonds differ from those obtained with issuance-contingent payments. Because one-period bonds may increase the government's exposure to rollover risk, differences in results are wider for the sudden-stop model, for which rollover risk is more significant. With sudden stops, lower default probabilities with one-period bonds are not only the result of the elimination of debt dilution but also the result of the lower debt levels chosen by the borrower to mitigate rollover risk.

	Baseline		Sudden stops	
	issuance-contingent	one-period	issuance-contingent	one-period
Defaults per 100 years	0.91	0.34	0.86	0.03
Mean debt (market value)	0.20	0.19	0.19	0.08
Mean debt (face value)	0.21	0.20	0.20	0.08
$E(R_s)$	0.95	0.44	0.52	0.15
$\sigma(R_s)$	0.68	0.45	0.65	0.27
$\sigma(y)$	3.28	3.10	3.25	3.22
$\sigma(c)/\sigma(y)$	1.10	1.23	1.11	1.28
$\rho(c, y)$	0.99	0.97	0.99	0.80
$\rho(R_s, y)$	-0.63	-0.73	-0.62	-0.41
Welfare gain (% of cons.)		0.13		-0.10

Table 4: Business cycle statistics without debt dilution.

In terms of welfare, while introducing issuance-contingent payments that eliminate dilution results in welfare gains when the government is subject to sudden-stop shocks, replacing long-duration bonds by one-period bonds results in welfare losses in that economy. The ex-ante welfare gain from introducing issuance-contingent payments is equivalent to a permanent consumption

increase of 0.12%, and the ex-ante welfare loss from issuing one-period bonds instead of long-duration bonds is equivalent to a permanent consumption decline of 0.8%.

4.5 An economy with state-contingent claims

Equation (6) makes clear that the compensation scheme that eliminates dilution introduces state-contingent payments. In this subsection we explore how that compensation scheme performs against the best possible scheme with one-period claims. The best market structure for the government would be one in which it can transfer resources across periods using contracts with payoffs that are conditional on the past history of state realizations. For tractability reasons, in this subsection we consider a market structure in which the government can issue one-period Arrow-Debreu securities that pay off conditional on the next-period domestic income realization. We assume that the government is subject to the same limited liability constraint that is present in the benchmark economy.¹⁷ The formal description of the model is presented in the Appendix.

By issuing a full range of state-contingent claims, the government can effectively eliminate defaults. For that reason, we cannot use the same criterion for selecting samples that has been used in previous tables. In order to facilitate the comparison of simulations across economies, we restrict to the same sample periods in the two economies. That is, the simulations in the economy with state-contingent claims are computed using the same 500 samples of 32 periods that were used to compute the simulations in the benchmark economy. The income shocks are the same, though the initial debt levels may differ.

Table 5 summarizes the simulation results. Mean debt levels take similar values in both economies, but the consumption process is more disentangled from the income process in the economy with state-contingent claims. The average ex-ante welfare gain from moving to an

¹⁷Issuing long-term debt allows the government to bring resources forward from future periods, not only from the subsequent period. That is not an option in the economy with one-period Arrow-Debreu securities. Clearly, this limitation is not an issue in the absence of the limited liability constraint. But it can dampen the welfare gain from issuing Arrow-Debreu securities in the presence of such constraint. However, we find that in the initial periods of our simulations (starting from a state with zero debt), consumption is lower on average in our benchmark economy with long-term debt than in the economy with one-period Arrow-Debreu securities.

	With dilution	Contingent claims
Defaults per 100 years	5.59	0
Mean debt market value	0.21	0.30
Mean debt face value	0.30	0.30
$\sigma(y)$	2.97	2.97
$\sigma(c)/\sigma(y)$	1.03	0.67
$\rho(c, y)$	1.00	0.87

Table 5: Business cycle statistics in the benchmark economy and in the economy with one-period Arrow-Debreu securities.

economy with state-contingent claims amounts to a permanent increase in consumption of 0.53%. This means that eliminating dilution contributes to 25% of the welfare gain that can be achieved by introducing state-contingent payment obligations.

4.6 Alternative issuance-contingent payments

Up to this point, we focused on issuance-contingent payments that eliminate dilution or on state-contingent payments. We show that these contingent payments could reduce default risk significantly and thus generate welfare gains. However, implementing the previously discussed contingent payments face several challenges. First, it requires knowledge of the state of the economy. That is simple in the model but it is significantly more difficult in practice. Second, even if one could narrow down the description of the state, it requires verifiability of the state realization. Finally, in the case of issuance-contingent payments that eliminate dilution, it requires knowledge of the counterfactual bond price that would have been observed absent new debt issuances.

In this subsection, we present results using two simple compensation schemes that depend on variables that may be easier to verify. We first study the case in which issuance-contingent payments are a predetermined fixed share of issuance revenues (i.e., a fixed share of $q(b', y)(\tilde{b} - b')$).

We search for the optimal share of issuance revenues the government should promise to existing bondholders and we find that this share is 120%.¹⁸ Table 6 presents the simulation results. With the optimal fixed-share payments the government could achieve 91% of the decline in the default frequency and 75% of the ex-ante welfare gain it achieves with the issuance-contingent payments that eliminate dilution. The main difference between fixed-share issuance-contingent payments and the issuance-contingent payments that eliminate dilution is that the former are an increasing function of the post-issuance bond price $q(b', y)$ while the latter are a decreasing function of $q(b', y)$.

	dilution	no dilution	fixed share	linear
Defaults per 100 years	5.59	0.91	1.71	0.81
Mean debt market value	0.21	0.20	0.17	0.19
Mean debt face value	0.30	0.21	0.20	0.20
$E(R_s)$	7.21	0.95	2.01	0.79
$\sigma(R_s)$	2.54	0.68	1.31	0.66
$\sigma(y)$	2.97	3.27	3.05	3.28
$\sigma(c)/\sigma(y)$	1.03	1.10	0.96	1.10
$\rho(c, y)$	1.00	0.99	0.99	0.99
$\rho(R_s, y)$	-0.81	-0.63	-0.64	-0.61
Welfare gain (% of cons.)		0.13	0.09	0.13

Table 6: Simulation results for different issuance-contingent payments.

Next, we study the effects of introducing issuance-contingent payments that are a decreasing function of the post-issuance bond price (and do not depend on the no-issuance bond price). In particular, we assume that the government promises to pay the holder of each existing bond $A - q(b', y)$ every time it issues debt. We search for the optimal value of A and find that this

¹⁸It should be mentioned that promising higher issuance-contingent payments need not necessarily be costly for the government because it allows the government to sell bonds at a higher price.

value is 1.75% higher than the price of a risk-free bond. Table 6 shows that this optimal issuance-contingent function would allow the government to achieve the same welfare gain than with the issuance-contingent payments that eliminate dilution, with a slightly lower default frequency and a slightly higher consumption volatility. In summary, our findings indicate that simple issuance-contingent payments could also reduce default risk significantly and be welfare enhancing.

5 Conclusions

We solved a baseline sovereign default framework à la Eaton and Gersovitz (1981) assuming that the sovereign promises that every time it issues debt it will pay to the holder of each existing bond the difference between the post-issuance bond price and the counterfactual no-issuance price. This issuance-contingent payment makes the value of a bond independent from future debt issuances and thus, it eliminates the debt dilution problem. We measured the effects of debt dilution by comparing the simulations of this model with the ones of the baseline model without issuance-contingent payments. We found that even without commitment to future re-payment policies and without optimally designed indexed bonds, if the sovereign could eliminate debt dilution, the default probability decreases 84%. We also showed that most gains from eliminating dilution can be obtained with issuance-contingent payments that are independent from the counterfactual bond price.

Analyzing debt dilution in a production economy would allow for a better quantification of the welfare gains from eliminating dilution. Several studies find evidence of a significant effect of interest rates on productivity (through the allocation of factors of production), and of a significant role of interest rate fluctuations in the amplification of shocks (see, for example, Mendoza and Yue (forthcoming), Neumeyer and Perri (2005), and Uribe and Yue (2006)). In light of these findings, our results indicate that the welfare cost of debt dilution may be large. The developing of a sovereign default framework that accommodates effects of interest rates on factors allocation is the subject of ongoing research (see, for example, Mendoza and Yue (forthcoming) and Sosa Padilla (2010)). An interesting extension of our analysis would be to study the implications of the debt dilution problem in such a framework.

Our findings indicate that governments could benefit from committing to lower future borrowing levels, which governments could achieve through fiscal rules. According to the IMF fiscal rules database (IMF (2009) describes this database), the number of countries with fiscal rules grew continuously from five in 1990 to eighty two in 2009. Our results indicate that eliminating debt dilution should be an important motivation for the implementation of fiscal rules that could reduce significantly the risk of debt crises and the mean and volatility of interest rates (see Hatchondo et al. (2011)). Fiscal crises are occurring in countries with fiscal rules in part because of the lack of enforcement of these rules. The covenants studied in this paper could help with the enforcement of fiscal rules by providing incentives for lower future borrowing levels. Implementing these covenants would necessitate the same strick accounting norms necessary for the successful implementation of fiscal rules.

As in Chatterjee and Eyigungor (forthcoming) and Hatchondo and Martinez (2009), we assumed that the government cannot choose the duration of its debt. Relaxing this assumption could enhance our understanding of the effects of debt dilution. It has been argued that the dilution problem may lead to excessive issuances of short-term debt (e.g., Kletzer (1984)). However, allowing the government to choose the duration of its debt would increase the computation cost significantly.¹⁹

¹⁹If one allows the government to choose a different duration of sovereign bonds each period, one would have to keep track of how many bonds the government has issued for each possible value of duration to determine government's liabilities (Arellano and Ramanarayanan (2010) study a model in which the government can choose to issue bonds with two possible durations). The computation cost of including additional state variables may be significant (Hatchondo et al. (2010) show that the computation cost of obtaining accurate solutions in default models may be significant, and Chatterjee and Eyigungor (forthcoming) explain how the cost increases when long-duration bonds are assumed).

References

- Aguiar, M. and Gopinath, G. (2006). ‘Defaultable debt, interest rates and the current account’. *Journal of International Economics*, volume 69, 64–83.
- Aguiar, M. and Gopinath, G. (2007). ‘Emerging markets business cycles: the cycle is the trend’. *Journal of Political Economy*, volume 115, no. 1, 69–102.
- Alfaro, L. and Kanczuk, F. (2009). ‘Optimal reserve management and sovereign debt’. *Journal of International Economics*, volume 77(1), 23–36.
- Alvarez, F. A., Marques, L. B., and Toledo, M. (2011). ‘Business Cycles in Emerging Markets: The Role of Durable Goods and Financial Frictions’. Manuscript, Universidad Carlos III.
- Amador, M. (2003). ‘A Political Economy Model of Sovereign Debt Repayment’. Manuscript, Stanford University.
- Arellano, C. (2008). ‘Default Risk and Income Fluctuations in Emerging Economies’. *American Economic Review*, volume 98(3), 690–712.
- Arellano, C. and Ramanarayanan, A. (2010). ‘Default and the Maturity Structure in Sovereign Bonds’. Mimeo, University of Minnesota.
- Asquith, P., Beatty, A., and Weber, J. (2005). ‘Performance pricing in bank debt contracts’. *Journal of Accounting and Economics*, volume 40, 101128.
- Athreya, K., Tam, X. S., and Young, E. R. (2007). ‘Does Rising Income Risk Lead to Better Risk Sharing?’ Manuscript, University of Virginia.
- Backus, D., Foresi, S., and Telmer, C. (1998). ‘Discrete-time models of bond pricing’. NBER Working Paper 6736.
- Bi, R. (2006). ‘Debt Dilution and the Maturity Structure of Sovereign Bonds’. Manuscript, University of Maryland.

- Bianchi, J. (forthcoming). ‘Overborrowing and Systemic Externalities in the Business Cycle’. *American Economic Review*.
- Bizer, D. S. and DeMarzo, P. (1992). ‘Sequential Banking’. *Journal of Political Economy*, volume 100, 41–61.
- Bolton, P. and Jeanne, O. (2009). ‘Structuring and Restructuring Sovereign Debt: The Role of Seniority’. *Review of Economic Studies*, volume 76, 879–902.
- Bolton, P. and Skeel, D. A. (2005). ‘Redesigning the International Lender of Last Resort’. *Chicago Journal of International Law*, volume 6, no. 1, 177–201.
- Borensztein, E., Chamon, M., Jeanne, O., Mauro, P., and Zettelmeyer, J. (2004). ‘Sovereign Debt Structure for Crisis Prevention’. IMF occasional paper 237.
- Borri, N. and Verdelhan, A. (2009). ‘Sovereign Risk Premia’. Manuscript, MIT.
- Boz, E. (2011). ‘Sovereign Default, Private Sector Creditors, and the IFIs’. *Journal of International Economics*, volume 83, 70–82.
- Boz, E., Daude, C., and Durdu, B. (2008). ‘Emerging Market Business Cycles Revisited: Learning about the Trend’. Board of Governors International Finance Discussion Papers No. 927.
- Broner, F. A., Lorenzoni, G., and Schukler, S. L. (2007). ‘Why do emerging economies borrow short term?’ National Bureau of Economic Research Working Paper 13076.
- Chatterjee, S., Corbae, D., Nakajima, M., and Ríos-Rull, J.-V. (2007). ‘A Quantitative Theory of Unsecured Consumer Credit with Risk of Default’. *Econometrica*, volume 75, 1525–1589.
- Chatterjee, S. and Eyigungor, B. (forthcoming). ‘Maturity, Indebtedness and Default Risk’. *American Economic Review*.
- Cruces, J. J., Buscaglia, M., and Alonso, J. (2002). ‘The Term Structure of Country Risk and Valuation in Emerging Markets’. Manuscript, Universidad Nacional de La Plata.

- Cruces, J. J. and Trebesch, C. (2011). ‘Sovereign Defaults: The Price of Haircuts’. CESIFO Working Paper No. 3604.
- Cuadra, G., Sanchez, J. M., and Sapriza, H. (forthcoming). ‘Fiscal policy and default risk in emerging markets’. *Review of Economic Dynamics*. Doi:10.1016/j.red.2009.07.002.
- Cuadra, G. and Sapriza, H. (2008). ‘Sovereign default, interest rates and political uncertainty in emerging markets’. *Journal of International Economics*, volume 76, 7888.
- D’Erasmus, P. (2008). ‘Government Reputation and Debt Repayment in Emerging Economies’. Manuscript, University of Texas at Austin.
- Detragiache, E. (1994). ‘Sensible buybacks of sovereign debt’. *Journal of Development Economics*, volume 43, 317–333.
- Durdu, C. B., Mendoza, E. G., and Terrones, M. E. (2009). ‘Precautionary demand for foreign assets in Sudden Stop economies: An assessment of the New Mercantilism’. *Journal of Development Economics*, volume 89(2), 194–209.
- Durdu, C. B., Nunes, R., and Sapriza, H. (2010). ‘News and sovereign default risk in small open economies’. International Finance Discussion Papers 997, Board of Governors of the Federal Reserve System.
- Eaton, J. and Fernandez, R. (1995). ‘Sovereign debt’. NBER Working Paper No. 5131.
- Eaton, J. and Gersovitz, M. (1981). ‘Debt with potential repudiation: theoretical and empirical analysis’. *Review of Economic Studies*, volume 48, 289–309.
- Eichengreen, B., Gupta, P., and Mody, A. (2008). ‘Sudden Stops and IMF-Supported Programs’. In *Financial Markets Volatility and Performance in Emerging Markets*, Sebastian Edwards and Mrcio G. P. Garcia, editors, University of Chicago Press.
- G-10 (2002). ‘Report of the G-10 Working Group on Contractual Clauses’.

- González-Rozada, M. and Levy Yeyati, E. (2008). ‘Global factors and emerging market spreads’. *Economic Journal*, volume 118, 1917-1936.
- Hatchondo, J. C. and Martinez, L. (2009). ‘Long-duration bonds and sovereign defaults’. *Journal of International Economics*, volume 79, 117-125.
- Hatchondo, J. C., Martinez, L., and Roch, F. (2011). ‘Fiscal rules and the sovereign default premium’. Mimeo, University of Chicago.
- Hatchondo, J. C., Martinez, L., and Saprizza, H. (2009). ‘Heterogeneous borrowers in quantitative models of sovereign default.’ *International Economic Review*, volume 50, 1129–1151.
- Hatchondo, J. C., Martinez, L., and Saprizza, H. (2010). ‘Quantitative properties of sovereign default models: solution methods matter’. *Review of Economic Dynamics*, volume 13, no. 4, 919–933.
- IMF (2002). ‘The Design and Effectiveness of Collective Action Clauses’.
- IMF (2003). ‘Proposed Features of a Sovereign Debt Restructuring Mechanism’.
- Kletzer, K. M. (1984). ‘Asymmetries of Information and LDC Borrowing with Sovereign Risk’. *The Economic Journal*, volume 94, 287–307.
- Krueger, A. O. and Hagan, S. (2005). ‘Sovereign Workouts: an IMF Perspective’. *Chicago Journal of International Law*, volume 6, no. 1, 203–218.
- Krusell, P. and Smith, A. (2003). ‘Consumption-savings decisions with quasi-geometric discounting’. *Econometrica*, volume 71, no. 1, 365–375.
- Li, W. and Sarte, P.-D. (2006). ‘U.S. consumer bankruptcy choice: The importance of general equilibrium effects’. *Journal of Monetary Economics*, volume 53, 613–631.
- Livshits, I., MacGee, J., and Tertilt, M. (2008). ‘Consumer Bankruptcy: A Fresh Start.’. *American Economic Review*, volume 97, 402–418.

- Lizarazo, S. (2005). ‘Sovereign risk and risk averse international investors’. Working Paper, ITAM.
- Lizarazo, S. (2006). ‘Contagion of financial crises in sovereign debt markets’. Working Paper, ITAM.
- Longstaff, F. A., Pan, J., Pedersen, L. H., and Singleton, K. J. (2007). ‘How Sovereign is Sovereign Credit Risk?’ NBER Working Paper No. 13658.
- Mendoza, E. and Yue, V. (forthcoming). ‘A General Equilibrium Model of Sovereign Default and Business Cycles’. *The Quarterly Journal of Economics*.
- Neumeyer, P. and Perri, F. (2005). ‘Business cycles in emerging economies: the role of interest rates’. *Journal of Monetary Economics*, volume 52, 345–380.
- Niepelt, D. (2008). ‘Debt Maturity without Commitment’. Manuscript, Study Center Gerzensee.
- Paulus, C. G. (2002). ‘A Legal Order for Insolvencies of States’. Working Paper, Humboldt University.
- Rodgers, C. (1965). ‘The corporate trust indenture project’. *Business Lawyer*, volume 20, 551–571.
- Sachs, J. and Cohen, D. (1982). ‘LDC borrowing with default risk’. National Bureau of Economic Research Working Paper 925.
- Sanchez, J. M. (2010). ‘The IT Revolution and the Unsecured Credit Market’. Federal Reserve Bank of St. Louis Working Paper.
- Saravia, D. (2010). ‘On The Role and Effects of IMF Seniority’. *Journal of International Money and Finance*, volume 29(6), 1024–1044.
- Smith, C. W. and Warner, J. B. (1979). ‘On Financial Contracting: An Analysis of Bond Covenants’. *Journal of Financial Economics*, volume 7, 117–161.

- Sosa Padilla, C. (2010). ‘Sovereign Defaults and Banking Crises’. Manuscript, University of Maryland.
- Sturzenegger, F. and Zettelmeyer, J. (2006). ‘Defaults in the 90s’. Manuscript, Universidad Torcuato Di Tella.
- Tirole, J. (2002). ‘Financial Crises, Liquidity, and the International Monetary System’. Princeton University Press.
- UN (2004). ‘Strategic issues in managing sovereign debt for sustained development: an issues paper for the multi-stakeholder dialogue on debt’. United Nations.
- Uribe, M. and Yue, V. (2006). ‘Country spreads and emerging countries: Who drives whom?’ *Journal of International Economics*, volume 69, 6–36.
- Vasicek, O. (1977). ‘An equilibrium characterization of the term structure’. *Journal of Financial Economics*, volume 5, 177–188.
- Yue, V. (2010). ‘Sovereign default and debt renegotiation’. *Journal of International Economics*, volume 80, no. 2, 176–187.

A An extension with one-period state-contingent claims

This appendix presents the case in which the government issues one-period state contingent claims. The borrowing protocol is as follows: i) at the beginning of the period the government announces how many contingent claims it wants to buy or issue, ii) lenders offer a price at which it is willing to trade each claim, and iii) the government trades claims with the lender that offers the best prices. The dynamic programming problem faced by the government can be summarized in the following functional equations

$$W(b, y) = \max_{d \in \{0,1\}} \{d\tilde{W}(1, b, y) + (1-d)\tilde{W}(0, b, y)\}$$

where

$$\begin{aligned} \tilde{W}(d, b, y) &= \max_{b'(y')} \left\{ u(c) + \beta \int W(b'(y'), y') F(dy' | y) \right\} \\ \text{s.t. } &c = y - d\phi(y) + (1-d)b - e^{-r} \int b'(y') dy' \\ &b'(y') \geq \underline{b}(y') \text{ for all } y' \end{aligned}$$

The borrowing limit $\underline{b}(y')$ satisfies

$$\tilde{W}(1, \underline{b}(y), y) = \tilde{W}(0, \underline{b}(y), y) \text{ for all } y$$

which, given the default cost assumed in the paper, implies $\underline{b}(y) = -\phi(y)$.

That is, lenders would offer to buy up to $-\underline{b}(y')$ claims that pay contingent on next-period income taking the value y' . Lenders buy those claims at the risk free price e^{-r} . If the government were to choose $b'(y) < \underline{b}(y')$, it would default when next period income takes the value y' and, thus, lenders would offer a zero price for those claims.

Figure 3 illustrates how the model works. The government would want to smooth out income shocks by borrowing more heavily against high income realization than against low income realizations. But its borrowing decisions are constrained by the limited liability constraint.

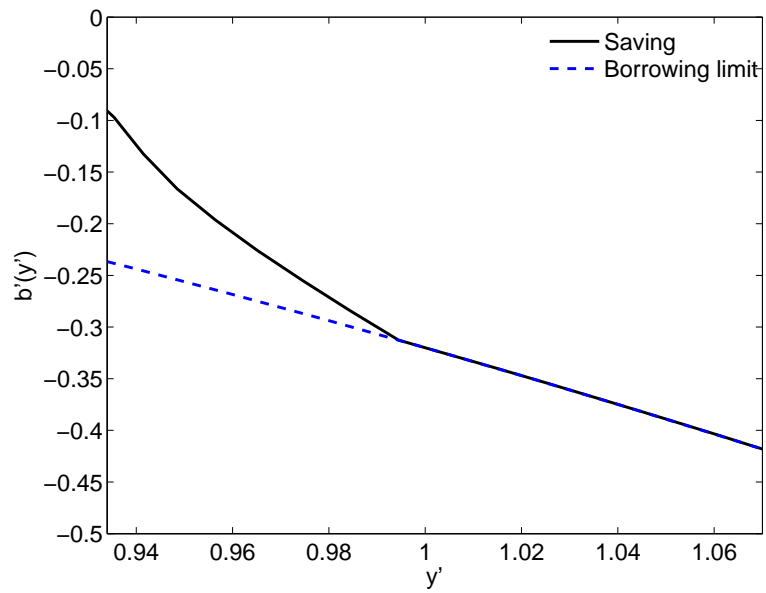


Figure 3: Saving decision when the initial savings equal the average savings observed in the simulations (-0.30) and the initial income equals the unconditional mean of the income distribution.