Local Mate Competition (LMC). Here we are interested in the question: how does the number of mates affect the proportion of resources that should be allocated by a hermaphrodite to male and females function in order to maximize individual fitness?

DEFINING THE VARIABLES...

a_i = allocation to male function by the ith hermaphrodite in the population.

a = average allocation to male function in the population (this was a* in lecture, but I can't use the star here, because it is used exclusively by Mathematica as the multiplication symbol.)

R = Total resource base for reproduction

k = the number of mates

V = The reproductive value though male fuction

W_m = Fitness through male function

W_f = Fitness though female function (assuming Bateman's principle).

W_i = Fitness of the ith individual (Wf + Wm)

FIRST WE DEFINE FITNESS THROUGH FEMALE FUNCTION, W_f.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN." This will store W_f in memory)

```
Wf = R*(1 - ai)
```

```
(1 - ai) R
```

NEXT DEFINE FITNESS THROUGH MALE FUNCTION, W_m. V is the reproductive value of male function, which will be defined shortly.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
Wm = V*R*ai
```

$$ai\,R\,V$$

NOW DEFINE THE VARIABLE, V, AS THE RATIO OF EGGS IN THE POPULATION TO SPERM IN THE POPULATION.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
V = (k*(1 - a))/((k - 1)*a + ai)
```

$$\frac{(1-a)\,k}{ai + a\,(-1+k)}$$

TOTAL FITNESS OF THE INDIVIDUAL IS THE SUM OF FITNESS THROUGH FEMALE + MALE FUNCTION.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
Wi = Wf + Wm
```

$$(1-ai)\,R + \frac{(1-a)\,ai\,k\,R}{ai + a\,(-1+k)}$$

NOW CALCULATE THE FIRST DERIVATIVE OF INDIVIDUAL FITNESS WITH RESPECT TO ALLOCATION TO MALE FUNCTION.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
Firstder = D[Wi, ai]
```

$$-R - \frac{(1-a)\ ai\ k\ R}{(ai + a\ (-1+k))^2} + \frac{(1-a)\ k\ R}{ai + a\ (-1+k)}$$

CALCULATE THE SECOND DERIVATIVE OF INDIVIDUAL FITNESS AS A FUNCTION OF ITS ALLOCATION TO MALE FUNCTION. Remember, local stability requires a negative second derivative

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
Secondder = D[Firstder, ai]
```

$$\frac{2\ (1-a)\ ai\ k\ R}{(ai + a\ (-1+k))^3} - \frac{2\ (1-a)\ k\ R}{(ai + a\ (-1+k))^2}$$

SIMPLIFY THE SECOND DERIVATIVE.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
Simplify [Secondder]
```

$$\frac{2\ (-1+a)\ a\ (-1+k)\ k\ R}{(ai + a\ (-1+k))^3}$$

Note that the second derivative is less that zero when the numerator is negative, which is true whenever $k > 1$ AND $a < 1$. Hence for $k > 1$ (i.e., more than one mate) and $a < 1$ (i.e., less than 100% allocation to male function in the population), we can find a locally stable ESS by setting $a_i = a$ and solving for the case when the first derivative equal to zero.

SET $a_i = a$. REMEMBER WHEN THE POPULATION IS AT THE ESS, INDIVIDUAL FITNESS SHOULD BE MAXIMIZED AT THE ESS.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
ai = a
```

```
a
```

SET THE FIRST DERIVATIVE EQUAL TO ZERO AND SOLVE FOR a.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
Solve [Firstder == 0, a]
```

$$\left\{\left\{a \to \frac{-1+k}{-1+2\,k}\right\}\right\}$$

Hence the allocation to male function, a, at

the ESS is:

a = (k-1)/(2k - 1)

Now set the population allocation, a, at the ESS

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
a = (k - 1)/(2*k -1)
```

$$\frac{-1+k}{-1+2\,k}$$

Now let the individual allocation, ai, be a variable again. So (ai) is no longer equal to (a). The purpose is to graph individual fitness as a function of ai when the population is at the ESS. We would expect that Wi is maximized under this condition when ai converges on a.

I'm also going to set the Resources equal to 1 unit.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
ai = .
R=1
```

```
1
```

Now, for the crux of the bisquit. Let there be two mates (k = 2).

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
k=2
```

```
2
```

The ESS can be found for two mates by simplifying the population allocation, a.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
Simplify [a]
```

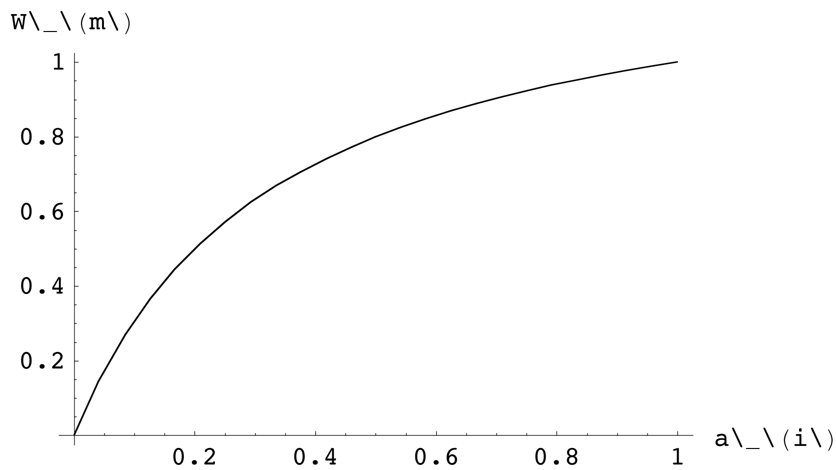
$$\frac{1}{3}$$

Hence, the ESS is at 0.333.

Now, let's plot the relationship between fitness through male function and the allocation by the ith individual to male function.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
Plot [{Wm}, {ai, 0, 1},
AxesLabel->{"ai","Wm"}]
```



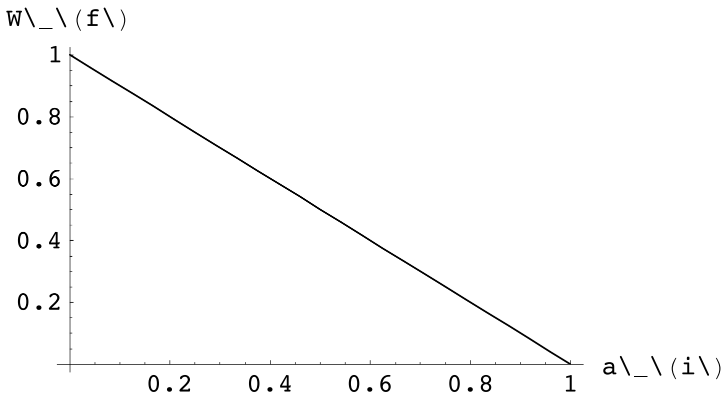

```
- Graphics -
```

Right! Diminishing returns, just as expected.

Now, let's plot the relationship between fitness through <u>female</u> function and the allocation by the ith individual to male function.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
Plot [{Wf}, {ai, 0, 1},
AxesLabel->{"ai","Wf"}]
```



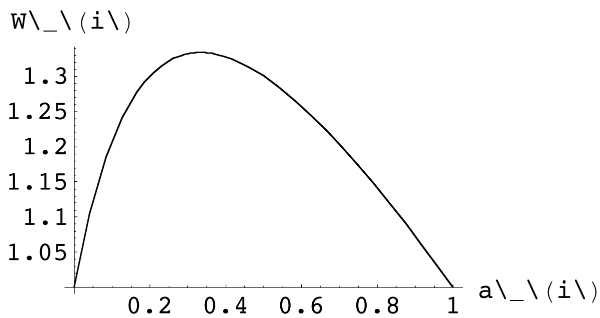

```
- Graphics -
```

Right! Linear increase in fitness with increasing allocation to female function. (This is a consequence of our assumtion that female function is limited only by resources.) REMEMBER: allocation to female fitness increases from right to left; it's equal to 1 - ai.

Bueno, Now let's plot the relationship between total fitness and the allocation by the ith individual to male function. It should be hump shaped.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
Plot [{Wi}, {ai, 0, 1},
AxesLabel->{"a_i","W_i"}]
```




```
- Graphics -
```

Note that individual fitness, Wi, is maximal when a_i = a = 0.33. That is when the individual allocation is at the ESS.

Now let's let the local mating population be large, say 100 mates.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
k=100
```

```
100
```

The new ESS for 100 mates is found by simplifying the parameter, a.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
Simplify [a]
```

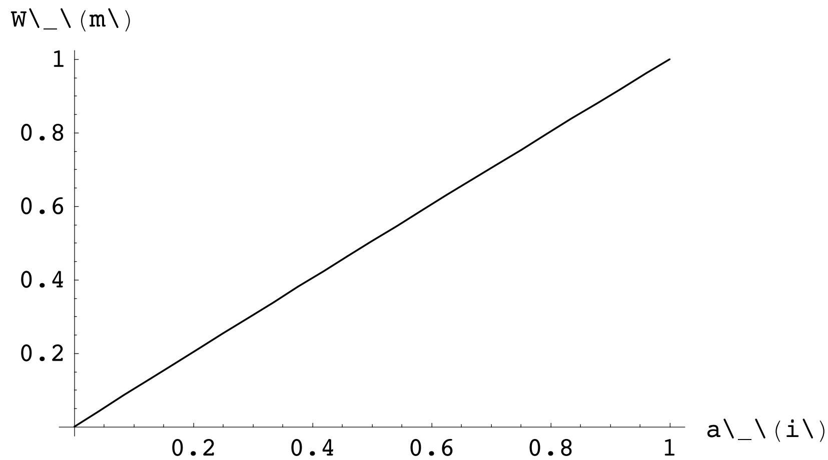
$$\frac{99}{199}$$

Hence, the ESS is at 99/199, which is about 1/2.

Now, as previously, let's first plot the relationship between fitness through male function and the allocation by the ith individual to male function.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
Plot [{Wm}, {ai, 0, 1},
AxesLabel->{"ai","Wm"}]
```



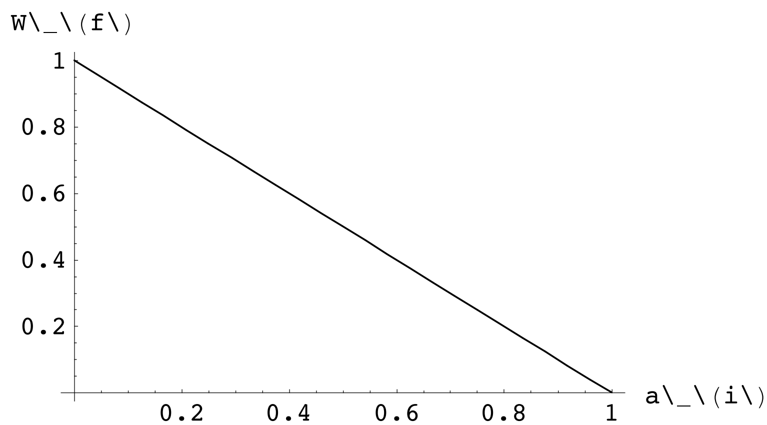

```
- Graphics -
```

Yes indeed. Virtually linear gains through male function when the local mating population is large.

Now, let's plot the relationship between fitness through <u>female</u> function and the allocation by the ith indiviudal to male function.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
Plot [{Wf}, {ai, 0, 1},
AxesLabel->{"a_i","W_f"}]
```



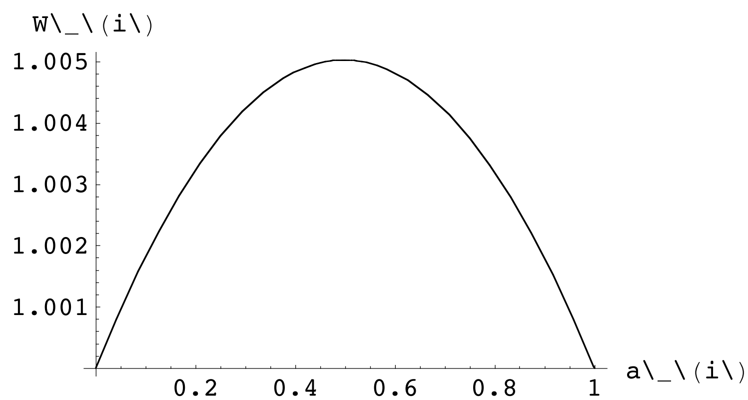

```
- Graphics -
```

Right! Linear increase in fitness with increasing allocation to female function. (This is a consequence of our assumtion that female function is limited only by resources.) REMEMBER: allocation to female fitness increases from right to left; it's equal to 1 - ai.

Finally, let's plot the relationship between total fitness and the allocation by the ith individual to male function.

(Put the cursor anywhere in the bracket below and press "SHIFT-RETURN.")

```
Plot [{Wi}, {ai, 0, 1},
AxesLabel->{"ai","Wi"}]
```




```
- Graphics -
```

Note that individual fitness, W_i, is maximal when $a_i = a = 0.50$.

If you are wondering why the surface is humped shaped, good for you! But notice the y axis. Compared to the previous hump for k = 2, this hump is virtually flat.

Now, set k to whatever you want and run through the same steps. Simply type in a number after the "k=" sign. You can then return to this point, change k to a new value and go again. Have fun.

```
k =  50
```

```
50
```

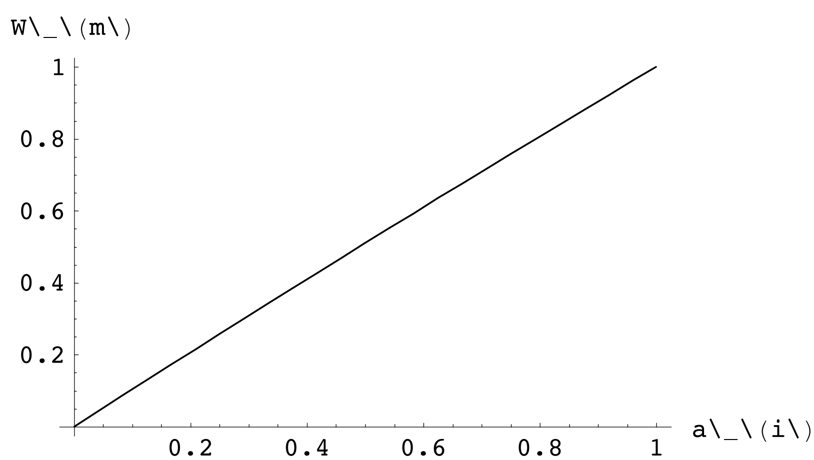
The new ESS for k mates is found by simplifying the parameter, a.

```
Simplify [a]
```

$$\frac{49}{99}$$

Plot the relationship between fitness through male function and the allocation by the ith individual to male function.

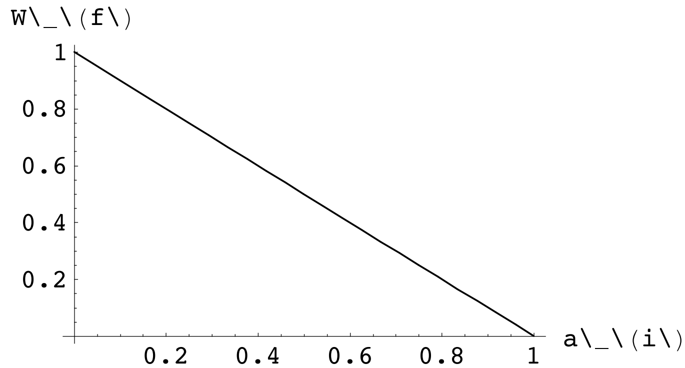
```
Plot [{Wm}, {ai, 0, 1},
AxesLabel->{"ai","Wm"}]
```




```
- Graphics -
```

Now, plot the relationship between fitness through female function and the allocation by the ith indiviudal to male function.

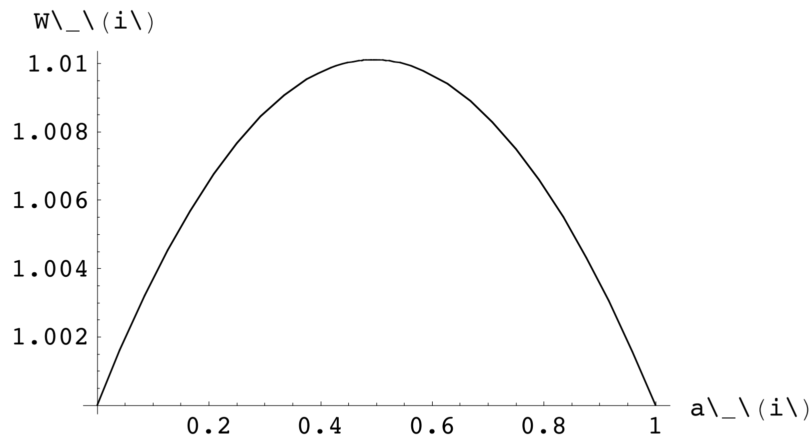
```
Plot [{Wf}, {ai, 0, 1},
AxesLabel->{"ai","Wf"}]
```




```
- Graphics -
```

Finally, plot the relationship between total fitness and the allocation by the ith individual to male function.

```
Plot [{Wi}, {ai, 0, 1},
AxesLabel->{"ai","Wi"}]
```




```
- Graphics -
```

Is the maximum fitness at the ESS?

Go back to the top of the Red text to run the program again.

Please close the application *Mathematica* when you are finished. This will reset the variables for the next person.

```
k=.
a=.
ai=.
R=.
V=.
```